

NOTES 7-3 LOGARITHMIC FUNCTIONS

**** $2^y = 4$ and $2^y = 8$ can be solved by guess/check, but to solve $2^y = 6$, logarithms are needed – more on that later.



Definition of Logarithm of Base a

$$a > 0$$

$$a \neq 1$$

**** $\log_a x = y$ means $a^y = x$
(the answer in a logarithm is an exponent)

Rewriting Exponential and Logarithmic Equations

Logarithmic form

$$\log_c w = r$$

$$\log_f g = k$$

$$\log_b a = t$$

Exponential form

$$c^r = w$$

$$f^k = g$$

$$b^t = a$$

Evaluating logs

answer

$$\log_2 16 = \boxed{4} \quad \text{because}$$

$$\log_{10} 10 = \boxed{1} \quad \text{because}$$

$$\log_3 1 = \boxed{0} \quad \text{because}$$

$$\log_6 216 = \boxed{3} \quad \text{because}$$

$$\log_8 64 = 2$$

$$\log_{13} 169 =$$

$$\log_5 \frac{1}{5} = -1$$

work

$$2^{\textcircled{4}} = 16$$

$$10^{\textcircled{?}} = 10$$

$$3^? = 1$$

$$\textcircled{3}^6 = 216$$

$$8^2 = 64$$

$$5^{\boxed{-1}} = \frac{1}{5}$$

1. $\log_2 16 = 4$

2. The log base 2 of 16 is 4.

3. The log base 2 of 16 is 4 because $2^4 = 16$.

1. $\log_{\square} \square = \square$

2. The log base of is .

3. The log base of is because .

1. $\log_{\square} \square = \square$

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1. $\log_2 16 = 4$
2. The log base 2 of 16 is 4.
3. The log base 2 of 16 is 4 because $2^4 = 16$.

1. $\log_{\boxed{3}} \boxed{9} = \boxed{2}$
2. The log base 3 of 9 is 2.
3. The log base 3 of 9 is 2 because $3^2 = 9$.

1. $\log_{\boxed{12}} \boxed{144} = \boxed{2}$
2. The log base 12 of 144 is 2.
3. The log base 12 of 144 is 2 because $12^2 = 144$.

1. $\log_2 16 = 4$
2. The log base 2 of 16 is 4.
3. The log base 2 of 16 is 4 because $2^4 = 16$.

1. $\log_5 125 = 3$
2. The log base 5 of 125 is 3.
3. The log base 5 of 125 is 3 because $5^3 = 125$.

1. $\log_6 \frac{1}{36} = -2$
2. The log base 6 of $\frac{1}{36}$ is -2.
3. The log base 6 of $\frac{1}{36}$ is -2 because $6^{-2} = \frac{1}{36}$.

1. $\log_2 16 = 4$
2. The log base 2 of 16 is 4.
3. The log base 2 of 16 is 4 because $2^4 = 16$.

1. $\log_{\boxed{8}} \boxed{4} = \boxed{\frac{2}{3}}$
2. The log base 8 of 4 is $\frac{2}{3}$.
3. The log base 8 of 4 is $\frac{2}{3}$ because $8^{\frac{2}{3}} = 4$.

1. $\log_{\boxed{36}} \boxed{\frac{1}{6}} = \boxed{-\frac{1}{2}}$
2. The log base 36 of $\frac{1}{6}$ is $-\frac{1}{2}$.
3. The log base 36 of $\frac{1}{6}$ is $-\frac{1}{2}$ because $36^{-\frac{1}{2}} = \frac{1}{6}$.

answer

$$\log_2 16 =$$

$$\log_3 27 =$$

$$\rightarrow \log_{16} 4 = \frac{1}{2}$$

$$\log_{\frac{1}{2}} 16 = -4$$

$$\log_9 3 =$$

$$\rightarrow \log_{\sqrt{5}} 25 = 4$$

work

$$16^{\square} = 4$$

$$16^x = 4 \quad \left(\frac{1}{2}\right)^x = 16$$

$$\begin{aligned} (\sqrt{5})^x &= 25 \\ (5^{\frac{1}{2}x}) &= 5^2 \end{aligned}$$

$$\begin{aligned} \frac{1}{2}x &= 2 \\ x &= 4 \end{aligned}$$

Common log – uses base 10 (and is a calculator button), but the base is not written

$$\log 1000 = 3$$

$$\text{because } 10^{\boxed{3}} = 1000$$

$$\log .01 = -2$$

$$\text{because } 10^{\boxed{-2}} = .01 = \frac{1}{100}$$

PRACTICE Evaluate the logarithms.

1. $\log_4 16$

2. $\log_5 1$

3. $\log 100$

$$\boxed{2}$$

4. $\log_4 2$

5. $\log_3 (-1)$ *undefined*

6. ~~$\log_4 4^{x+2}$~~

$$3^{\boxed{\quad}} = -1$$

$$4^{\boxed{\quad}} = 4^{x+2}$$

Domain
(input) $x > 0$
 $(0, \infty)$

$$\boxed{x+2}$$

Solve. (Rewrite in exponent form first.)

$$\log_4 x = 2$$

$$4^2 = x$$

$$\boxed{16} = x$$

$$\log_x 243 = 5$$

$$(x^5)^{1/5} = (243)^{1/5}$$

$$x = \boxed{3}$$

$$\log_5 (x - 4) = 0$$

$$5^0 = x - 4$$

$$1 = x - 4$$

$$x = \boxed{5}$$

$$5.) \log_{\frac{1}{2}} 4 = \underline{-2}$$

$$\left(\frac{1}{2}\right)^x = \frac{4}{1}$$

$$10.) \log_x \frac{1}{64} = -3$$

$$\left(x^{-3}\right)^{\frac{1}{3}} = \left(\frac{1}{64}\right)^{-\frac{1}{3}}$$

$$\boxed{\frac{1}{4}}^3 = \frac{1}{64}$$

flip

$$x = \boxed{4}$$

$$32.) \log_{x+2} 16 = 2$$

$${}^2\sqrt{(x+2)^2} = {}^2\sqrt{16}$$

$$x+2 = \pm 4$$

$$x+2 = 4$$

$$x = 2$$

~~$$x+2 = -4$$~~

~~$$x = -6$$~~

base: $-6+2 = -4$
can't be neg.