

LESSON

Reteach

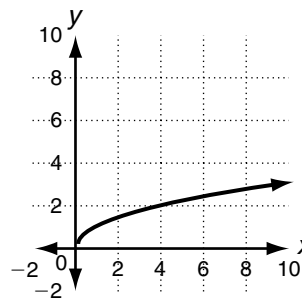
8-7 Radical Functions

The square root function, $f(x) = \sqrt{x}$, is a radical function.

The domain of $f(x) = \sqrt{x}$ is $\{x | x \geq 0\}$.

The range is $\{y | y \geq 0\}$.

Note that x and y have only nonnegative values.



You can make a table of values to graph a radical function.

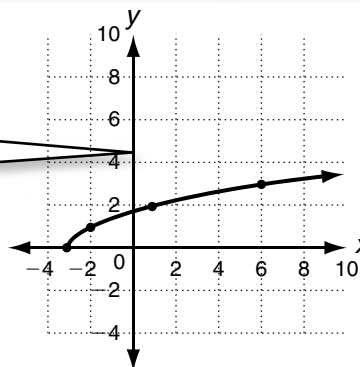
Graph: $f(x) = \sqrt{x + 3}$

x	$f(x) = \sqrt{x + 3}$	$(x, f(x))$
-3	$f(-3) = \sqrt{-3 + 3} = \sqrt{0} = 0$	$(-3, 0)$
-2	$f(-2) = \sqrt{-2 + 3} = \sqrt{1} = 1$	$(-2, 1)$
1	$f(1) = \sqrt{1 + 3} = \sqrt{4} = 2$	$(1, 2)$
6	$f(6) = \sqrt{6 + 3} = \sqrt{9} = 3$	$(6, 3)$

First choose the value of x that makes $f(x) = 0$.

First choose the value of x that make perfect squares.

The domain is $\{x | x \geq -3\}$.
The range is $\{y | y \geq 0\}$.



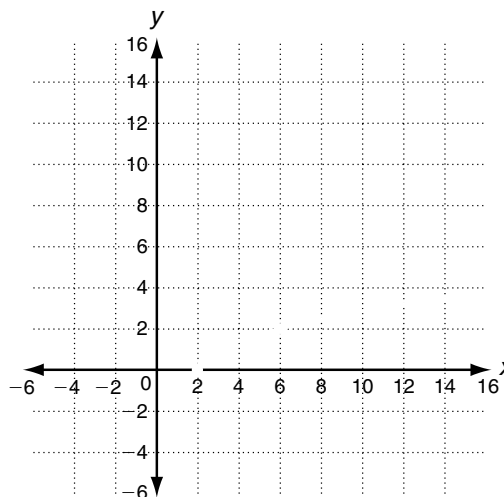
Graph the function. Identify its domain and range.

1. $f(x) = \sqrt{x - 2}$

x	$f(x) = \sqrt{x - 2}$	$(x, f(x))$
2		
3		

Domain: _____

Range: _____

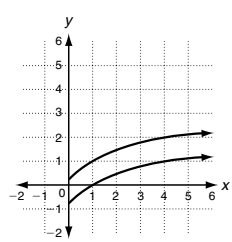
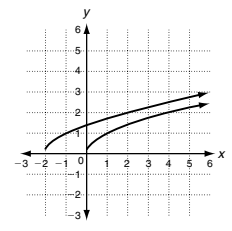
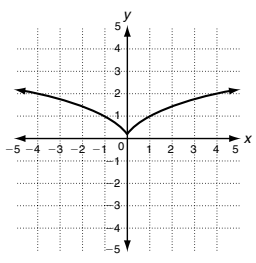


LESSON

Reteach

8-7 Radical Functions (continued)

Transformations of the square root function, $f(x) = \sqrt{x}$, are similar to transformations of other functions.

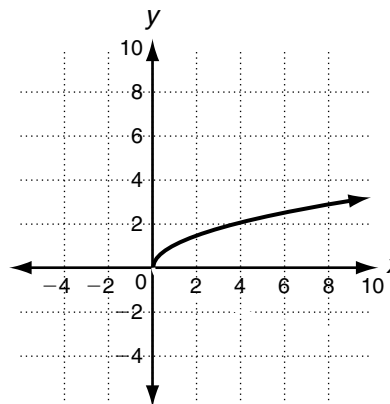
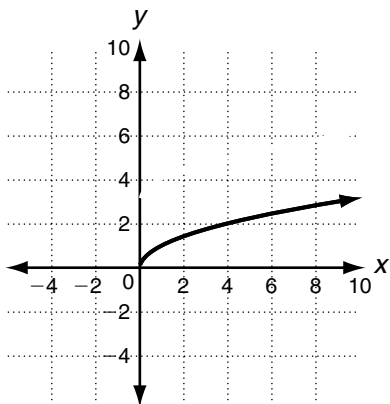
Transformations	
<p>Vertical Translations</p> <p>$y = \sqrt{x} + k$</p> <p>Shifts $f(x)$ up k units for $k > 0$</p> <p>Shifts $f(x)$ down k units for $k < 0$</p>	 <div style="border: 1px solid black; padding: 5px; margin-top: 10px;"> <p>$k = -1$, so $g(x) = \sqrt{x} - 1$ shifts $f(x)$ 1 unit down.</p> </div>
<p>Horizontal Translations</p> <p>$y = \sqrt{x - h}$</p> <p>Shifts $f(x)$ right h units for $h > 0$</p> <p>Shifts $f(x)$ left h units for $h < 0$</p>	 <div style="border: 1px solid black; padding: 5px; margin-top: 10px;"> <p>$h = -2$, so $m(x) = \sqrt{x + 2}$ shifts $f(x)$ 2 units left.</p> </div>
<p>Reflections</p> <p>$y = -\sqrt{x}$ reflects $f(x)$ across x-axis</p> <p>$y = \sqrt{-x}$ reflects $f(x)$ across y-axis</p>	 <div style="border: 1px solid black; padding: 5px; margin-top: 10px;"> <p>$r(x) = \sqrt{-x}$ reflects $f(x)$ across the y-axis.</p> </div>

Using the graph of $f(x) = \sqrt{x}$ as a guide, describe the transformation and graph each function.

2. $s(x) = \sqrt{x} + 3$

3. $p(x) = -\sqrt{x}$

$k = 3$: _____

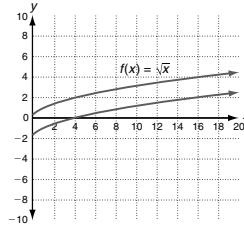


LESSON **Practice A**
8-7 Radical Functions

Graph each function.

1. $g(x) = \sqrt{x} - 2$

x	g(x)	(x, g(x))
0	$\sqrt{0} - 2 = -2$	(0, -2)
1	$\sqrt{1} - 2 = -1$	(1, -1)
4	$\sqrt{4} - 2 = 0$	(4, 0)
9	$\sqrt{9} - 2 = 1$	(9, 1)
16	$\sqrt{16} - 2 = 2$	(16, 2)



a. Describe the transformation from the parent function.

Translation 2 units down

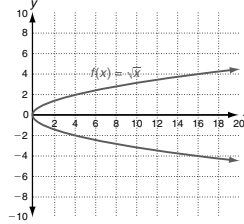
b. Identify the domain and range.

**Domain: $\{x \mid x \geq 0\}$;
range: $\{y \mid y \geq -2\}$**

2. $g(x) = -\sqrt{x}$

a. Complete the table of values, then graph.

x	g(x)	(x, g(x))
0	$-\sqrt{0} = 0$	(0, 0)
1	$-\sqrt{1} = -1$	(1, -1)
4	$-\sqrt{4} = -2$	(4, -2)
9	$-\sqrt{9} = -3$	(9, -3)
16	$-\sqrt{16} = -4$	(16, -4)



b. Describe the transformation from the parent function.

Reflection across the x-axis

c. Identify the domain and range.

**Domain: $\{x \mid x \geq 0\}$;
range: $\{y \mid y \leq 0\}$**

Solve.

3. Dale wants to horizontally stretch the function $f(x) = \sqrt{x+5}$ by a factor of 3. He writes the function $f(x) = \sqrt{3(x+5)}$. Is he correct? If not, what is the correct function?

No; $g(x) = \sqrt{\frac{1}{3}(x+5)}$

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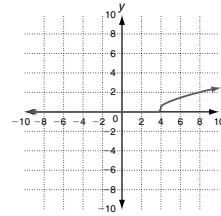
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LESSON **Practice B**
8-7 Radical Functions

Graph each function, and identify its domain and range.

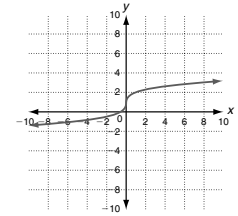
1. $f(x) = \sqrt{x-4}$



Domain: $\{x \mid x \geq 4\}$

Range: $\{y \mid y \geq 0\}$

2. $f(x) = \sqrt[3]{x} + 1$



Domain: **all real numbers**

Range: **all real numbers**

Using the graph of $f(x) = \sqrt{x}$ as a guide, describe the transformation.

3. $g(x) = 4\sqrt{x+8}$ **Vertical stretch by a factor of 4 and translate 8 units left**

4. $g(x) = -\sqrt{3x} + 2$ **Reflection across the x-axis, horizontal compression by a factor of $\frac{1}{3}$, and translate 2 units up**

Use the description to write the square root function g .

5. The parent function $f(x) = \sqrt{x}$ is reflected across the y-axis, vertically stretched by a factor of 7, and translated 3 units down.

$g(x) = 7\sqrt{-x} - 3$

6. The parent function $f(x) = \sqrt{x}$ is translated 2 units right, compressed horizontally by a factor of $\frac{1}{2}$, and reflected across the x-axis.

$g(x) = -\sqrt{2(x-2)}$

Solve.

7. For a gas with density, n , measured in atoms per cubic centimeter, the average distance, d , between atoms is given by $d = \left(\frac{3}{4\pi n}\right)^{\frac{1}{3}}$. The gas in a certain region of space has a density of just 10 atoms per cubic centimeter. Find the average distance between the atoms in that region of space.

0.29 cm

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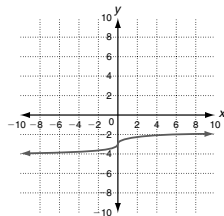
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LESSON **Practice C**
8-7 Radical Functions

Graph each function or inequality.

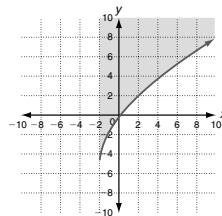
1. $g(x) = \frac{1}{2}\sqrt[3]{x} - 3$



a. Identify the domain and range.

**Domain: all real numbers;
range: all real numbers**

2. $y \geq 4\sqrt{x+2} - 6$



a. Describe the solution region.

The region above the curve including the line where $x \geq -2$

Use the description to write the square root function g .

3. The parent function $f(x) = \sqrt{x}$ is compressed vertically by a factor of $\frac{1}{4}$, reflected across the x-axis, and translated 6 units up.

$g(x) = -\frac{1}{4}\sqrt{x} + 6$

4. The parent function $f(x) = \sqrt{x}$ is translated 8 units left, reflected across the y-axis, and stretched horizontally by a factor of 3.

$g(x) = \sqrt{-\frac{1}{3}(x+8)}$

Solve.

5. The frequency, f , in Hz, at which a simple pendulum rocks back and forth is given by $f = \frac{1}{2\pi} \sqrt{\frac{g}{l}}$, where g is the strength of the gravitational field at the location of the pendulum, and l is the length of the pendulum.

a. Find the frequency of a pendulum whose length is 1 foot and where the gravitational field is approximately 32 ft/s².

0.90 Hz

b. The strength of the gravitational field on the moon is about $\frac{1}{6}$ as strong as on Earth. Find the frequency of the same pendulum on the moon.

0.37 Hz

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LESSON **Reteach**
8-7 Radical Functions

The square root function, $f(x) = \sqrt{x}$, is a radical function.

The domain of $f(x) = \sqrt{x}$ is $\{x \mid x \geq 0\}$.

The range is $\{y \mid y \geq 0\}$.

Note that x and y have only nonnegative values.

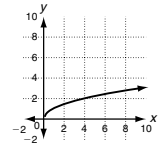
You can make a table of values to graph a radical function.

Graph: $f(x) = \sqrt{x+3}$

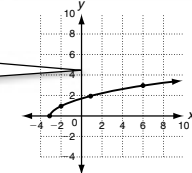
x	$f(x) = \sqrt{x+3}$	(x, f(x))
-3	$f(-3) = \sqrt{-3+3} = \sqrt{0} = 0$	(-3, 0)
-2	$f(-2) = \sqrt{-2+3} = \sqrt{1} = 1$	(-2, 1)
1	$f(1) = \sqrt{1+3} = \sqrt{4} = 2$	(1, 2)
6	$f(6) = \sqrt{6+3} = \sqrt{9} = 3$	(6, 3)

First choose the value of x that makes $f(x) = 0$.

First choose the value of x that make perfect squares.



The domain is $\{x \mid x \geq -3\}$.
The range is $\{y \mid y \geq 0\}$.



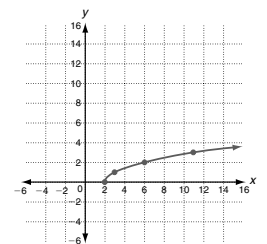
Graph the function. Identify its domain and range.

1. $f(x) = \sqrt{x-2}$

x	$f(x) = \sqrt{x-2}$	(x, f(x))
2	0	(2, 0)
3	1	(3, 1)
6	2	(6, 2)
11	3	(11, 3)

Domain: $\{x \mid x \geq 2\}$

Range: $\{y \mid y \geq 0\}$



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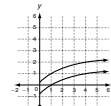
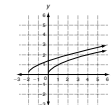
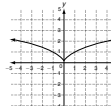
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LESSON **Reteach**

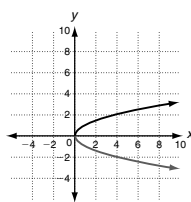
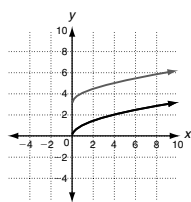
8-7 Radical Functions (continued)

Transformations of the square root function, $f(x) = \sqrt{x}$, are similar to transformations of other functions.

Transformations	
<p>Vertical Translations</p> $y = \sqrt{x} + k$ Shifts $f(x)$ up k units for $k > 0$ Shifts $f(x)$ down k units for $k < 0$	 <p>$k = -1$, so $g(x) = \sqrt{x} - 1$ shifts $f(x)$ 1 unit down.</p>
<p>Horizontal Translations</p> $y = \sqrt{x - h}$ Shifts $f(x)$ right h units for $h > 0$ Shifts $f(x)$ left h units for $h < 0$	 <p>$h = -2$, so $m(x) = \sqrt{x + 2}$ shifts $f(x)$ 2 units left.</p>
<p>Reflections</p> $y = -\sqrt{x}$ reflects $f(x)$ across x -axis $y = \sqrt{-x}$ reflects $f(x)$ across y -axis	 <p>$f(x) = \sqrt{-x}$ reflects $f(x)$ across the y-axis.</p>

Using the graph of $f(x) = \sqrt{x}$ as a guide, describe the transformation and graph each function.

2. $s(x) = \sqrt{x} + 3$
 $k = 3$: Shifts $f(x)$ 3 units up
3. $p(x) = -\sqrt{x}$
 Reflects $f(x)$ across the x -axis



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LESSON **Challenge**

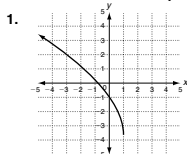
8-7 Graphs of Radical Functions

The first graph shows the function $y = \sqrt{x}$. Observe that both the domain and the range consist of the set of nonnegative numbers. The graph begins at the point $(0, 0)$ and includes points $(1, 1)$, $(4, 2)$, and $(9, 3)$. As x increases from 0 to 1, then from 1 to 4, and then from 4 to 9, the y value increases by 1 each time.

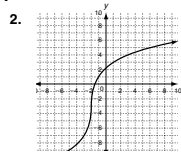
The second graph shows the function $y = \sqrt[3]{x}$. In this case the domain and range are both the set of real numbers. As x increases from 0 to 1, then from 1 to 8, and then from 8 to 27, the y -value increases by 1 each time.

Look at the third graph. You can determine the equation from the graph. The square root function has been reflected over the x -axis. The starting point is at $(2, 6)$ so both a reflection and a translation are involved. As x increases from the starting point 1 unit to the right, the y -value decreases 4 units so a vertical stretch is also indicated. Putting all these transformations together gives $y = -4\sqrt{x - 2} + 6$.

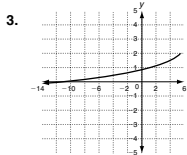
Use these observations to write an equation for each graph. Note that the equations may not be unique since many times a vertical stretch or compression can also be written using a horizontal stretch or compression, respectively.



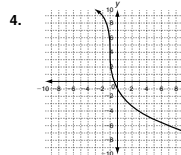
$y = 3\sqrt{-x + 1} - 4$



$y = 4\sqrt[3]{x + 2} - 3$



$y = -0.5\sqrt{-x + 5} + 2$



$y = 5\sqrt[3]{-x - 1} + 4$

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LESSON **Problem Solving**

8-7 Radical Functions

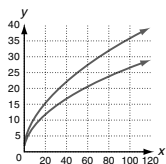
On Earth the distance, d , in kilometers that one can see to the horizon is a function of altitude, a , in meters, and can be found using the function $d(a) = 3.56\sqrt{a}$. To find the corresponding distance to the horizon on Mars, the function must be stretched horizontally by a factor of about $\frac{9}{5}$.

1. a. Write the function that corresponds to the given transformation.

$d(a) = 3.56\sqrt{\frac{5}{9}a}$

- b. Use a graphing calculator to graph the function and the parent function. Sketch both curves on the coordinate plane.

- c. Use your graph to determine the approximate distance to the horizon from an altitude of 100 meters:
 on Earth 36 km
 on Mars 27 km



Choose the letter for the best answer.

2. Which equation represents the radius of a sphere as a function of the volume of the sphere?
 A $r = \sqrt[3]{\frac{3\pi}{4V}}$ C $r = \sqrt[3]{\frac{4V}{3\pi}}$
 B $r = \sqrt[3]{\frac{3V}{4\pi}}$ D $r = \sqrt[3]{\frac{4\pi}{3V}}$
3. Alice graphed a function that is found only in the first quadrant. Which function could she have used?
 A $f(x) = \sqrt{x + 2}$ C $f(x) = \sqrt{x} + 2$
 B $f(x) = -\sqrt{x}$ D $f(x) = \sqrt{x} - 2$
4. Harry made a symmetrical design by graphing four functions, one in each quadrant. The graph of which function is in the third quadrant?
 A $f(x) = 4\sqrt{x}$ C $f(x) = -4\sqrt{x}$
 B $f(x) = 4\sqrt{-x}$ D $f(x) = -4\sqrt{-x}$
5. The side length of a cube can be represented by $s = \sqrt[3]{\frac{T}{6}}$, where T is the surface area of the cube. What transformation is shown by $s = \sqrt[3]{\frac{T}{3}}$?
 A Horizontal compression by a factor of 0.5
 B Horizontal stretch by a factor of 2
 C Vertical compression by a factor of 0.5
 D Vertical stretch by a factor of 2
6. The hypotenuse of a right isosceles triangle can be written $H = \sqrt{2x^2}$, where x is the length of one of the legs. Which function models the hypotenuse when the legs are lengthened by a factor of 2?
 A $H = \sqrt{2x^2} + 2$ C $H = \sqrt{4x^2}$
 B $H = \sqrt{2x^2} + 4$ D $H = \sqrt{8x^2}$

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LESSON **Reading Strategy**

8-7 Use a Visual Map

You can determine how a radical function has been reflected and translated on the coordinate plane by looking at its equation.

Parent Function	Transformed Function	
$f(x) = \sqrt{x}$	$f(x) = a\sqrt{\frac{1}{b}(x - h)} + k$	
	<table border="0"> <tr> <td>Reflection: $a < 0$ = across x-axis $b < 0$ = across y-axis</td> <td>Translation: $+h$ = right h units $-h$ = left h units $+k$ = up k units $-k$ = down k units</td> </tr> </table>	Reflection: $a < 0$ = across x -axis $b < 0$ = across y -axis
Reflection: $a < 0$ = across x -axis $b < 0$ = across y -axis	Translation: $+h$ = right h units $-h$ = left h units $+k$ = up k units $-k$ = down k units	

Identify the constants and describe the transformation from the parent function.

1. $g(x) = \sqrt{x} - 3$
 a. $a =$ 1 b. $b =$ 1
 c. $h =$ 0 d. $k =$ -3
 e. Describe the transformation. Translated 3 units down
2. $g(x) = -\sqrt{x - 1} + 2$
 a. $a =$ -1 b. $b =$ 1
 c. $h =$ 1 d. $k =$ 2
 e. Describe the transformation. Translated 2 units up, 1 unit right, and reflected across the x-axis
3. $g(x) = \sqrt{-x + 4} - 5$
 a. $a =$ 1 b. $b =$ -1
 c. $h =$ -4 d. $k =$ -5
 e. Describe the transformation. Translated 5 units down, 4 units left, and reflected across the y-axis
4. $g(x) = -\sqrt[3]{x + 3} + 2$
 a. $a =$ -1 b. $b =$ 1
 c. $h =$ -3 d. $k =$ 2
 e. Describe the transformation. Translated 2 units up, 3 units left, and reflected across the x-axis

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