Name ______________________ Date ___________ Class ___________

LESSON 8-3

Reteach

Adding and Subtracting Rational Expressions

Use a common denominator to add or subtract rational expressions.

Add: $\frac{6x + 4}{x + 5} + \frac{2x - 8}{x + 5}$.

Step 1 Add.

$$\frac{6x + 4}{x + 5} + \frac{2x - 8}{x + 5} = \frac{6x + 4 + 2x - 8}{x + 5}$$

The denominators are the same. Add the numerators.

$$= \frac{6x + 2x + 4 - 8}{x + 5}$$

Group like terms.

$$= \frac{8x - 4}{x + 5}$$

Combine like terms.

Step 2 Identify *x*-values for which the expression is undefined.

$x \neq -5$ because -5 makes the denominator equal 0.

Subtract: $\frac{4x - 3}{2x - 1} - \frac{8x + 2}{2x - 1}$.

Step 1 Subtract.

$$\frac{4x - 3}{2x - 1} - \frac{8x + 2}{2x - 1} = \frac{(4x - 3) - (8x + 2)}{2x - 1}$$

The denominators are the same. Subtract the numerators.

$$= \frac{4x - 3 - 8x - 2}{2x - 1}$$

Use the Distributive Property.

$$= \frac{-4x - 5}{2x - 1}$$

Combine like terms.

Step 2 Identify *x*-values for which the expression is undefined.

$x \neq \frac{1}{2}$ because $\frac{1}{2}$ makes the denominator equal 0.

Add or subtract.

1. $\frac{x - 5}{x^2 - 4} + \frac{3x + 2}{x^2 - 4}$

   $\frac{(x - 5) + (3x + 2)}{x^2 - 4}$

   ____

   $x \neq$ ______________________

2. $\frac{7x - 5}{x + 3} - \frac{4x - 1}{x + 3}$

   $\frac{(7x - 5) - (4x - 1)}{x + 3}$

   ____

   $x \neq$ ______________________

3. $\frac{2x - 1}{x - 1} - \frac{5x + 4}{x - 1}$

   ____

   ____

   $x \neq$ ______________________

4. $\frac{4x + 1}{3x + 7} + \frac{9 - x}{3x + 7}$

   ____

   $x \neq$ ______________________

5. $\frac{8 - x}{x - 3} - \frac{5 - x}{x - 3}$

   ____

   $x \neq$ ______________________

6. $\frac{5x + 2}{x^2 - 1} - \frac{3x - 7}{x^2 - 1}$

   ____

   $x \neq$ ______________________

Name ____________________ Date __________ Class __________

LESSON 8-3

Reteach

Adding and Subtracting Rational Expressions (continued)

Use the least common denominator (LCD) to add rational expressions with different denominators. The process is the same as adding fractions with different denominators.

Add: $\frac{x-4}{x^2+2x-3}+\frac{2x}{x-1}$.

Step 1 Factor denominators completely.

$$\frac{x-4}{x^2+2x-3}+\frac{2x}{x-1}=\frac{x-4}{(x+3)(x-1)}+\frac{2x}{x-1}$$

Step 2 Find the LCD.
The LCD is the least common multiple of the denominators:
$(x+3)(x-1)$.

Step 3 Write each term of the expression using the LCD.

$$\frac{2x}{x-1}=\frac{2x}{x-1}\left(\frac{x+3}{x+3}\right)=\frac{2x^2+6x}{(x-1)(x+3)}$$

So, $\frac{x-4}{(x+3)(x-1)}+\frac{2x}{x-1}=\frac{x-4}{(x+3)(x-1)}+\frac{2x^2+6x}{(x-1)(x+3)}$

Step 4 Add the numerators and simplify.

$$\frac{x-4+2x^2+6x}{(x+3)(x-1)}=\frac{2x^2+7x-4}{(x+3)(x-1)}$$

Step 5 Identify x-values for which the expression is undefined.

$x \neq -3$ or 1 because both values make the denominator equal 0.

Add.

7. $\frac{x-1}{x^2-4}+\frac{3x}{x+2}$

$\frac{x-1}{(x+2)(x-2)}+\frac{3x}{x+2}$

$\frac{x-1}{(x+2)(x-2)}+\frac{3x}{x+2}\left(\frac{x-2}{x-2}\right)$

____

$x \neq$ ____________________

8. $\frac{4x-1}{x^2+3x+2}+\frac{3}{x+1}$

____

____

____

$x \neq$ ____________________

9. What is the LCD of $\frac{2x+1}{x^2-9}$ and $\frac{7}{x^2-x-6}$?

____

LESSON 8-3

Practice A

Adding and Subtracting Rational Expressions

Add or subtract. Identify any x-values for which the expression is undefined.

1. $\frac{x}{x+1} + \frac{2x}{x+1}$ — $\frac{3x}{x+1}$; $x \neq -1$

2. $\frac{3x-1}{2x-5} - \frac{5x-2}{2x-5}$ — $\frac{-2x+1}{2x-5}$; $x \neq \frac{5}{2}$

Find the least common multiple for each pair.

3. $4x$ and $3x^2$ — $12x^2$

4. $(x+1)(x+2)$ and $(x+2)$ — $(x+1)(x+2)$

Add or subtract.

5. $\frac{4x+1}{x-4} + \frac{2x+7}{x-4}$ — $\frac{6x+8}{x-4}$

6. $\frac{6}{x} - \frac{2x}{x+2}$ — $\frac{-2x^2+6x+12}{x^2+2x}$

7. $\frac{x}{x^2-3x-4} + \frac{3}{x-4}$ — $\frac{4x+3}{x^2-3x-4}$

8. $\frac{x}{x^2-1} - \frac{2}{x+1}$ — $\frac{-x+2}{x^2-1}$

Simplify.

9. $\frac{\frac{x}{2}}{\frac{3}{x}}$ — $\frac{x^2}{6}$

10. $\frac{\frac{1}{x+5}}{\frac{x}{2}}$ — $\frac{2}{x^2+5x}$

11. $\frac{\frac{x+2}{3}}{\frac{3x+6}{9}}$ — 1

12. $\frac{\frac{x}{x+3}}{\frac{x+1}{5}}$ — $\frac{5x}{x^2+4x+3}$

Solve.

13. A ferry shuttles from Seattle to Vancouver Island and back. Because of headwinds, the return trip is slower than the trip to the island. The average speed of the ferry, in miles per hour, is given by the expression: $\frac{2d}{\frac{d}{50}+\frac{d}{60}}$.

What is the average speed of the ferry? **54.5 miles per hour**

LESSON 8-3

Practice B

Adding and Subtracting Rational Expressions

Find the least common multiple for each pair.

1. $3x^2y^6$ and $5x^3y^2$ — $15x^3y^6$

2. $x^2 + x - 2$ and $x^2 - x - 6$ — $(x-1)(x+2)(x-3)$

Add or subtract. Identify any x-values for which the expression is undefined.

3. $\frac{2x-3}{x+4} + \frac{4x-5}{x+4}$ — $\frac{6x-8}{x+4}$; $x \neq -4$

4. $\frac{x+12}{2x-5} - \frac{3x-2}{2x-5}$ — $\frac{-2x+14}{2x-5}$; $x \neq \frac{5}{2}$

5. $\frac{x+4}{x^2-x-12} + \frac{2x}{x-4}$ — $\frac{2x^2+7x+4}{x^2-x-12}$; $x \neq 4$, $x \neq -3$

6. $\frac{3x^2-1}{x^2-3x-18} - \frac{x+2}{x-6}$ — $\frac{2x^2-5x-7}{x^2-3x-18}$; $x \neq 6$, $x \neq -3$

7. $\frac{x+2}{x^2-2x-15} + \frac{x}{x+3}$ — $\frac{x^2-4x+2}{x^2-2x-15}$; $x \neq -3$, $x \neq 5$

8. $\frac{x+6}{x^2-7x-18} - \frac{2x}{x-9}$ — $\frac{-2x^2-3x+6}{x^2-7x-18}$; $x \neq -2$, $x \neq 9$

Simplify. Assume all expressions are defined.

9. $\frac{\frac{x-1}{x+5}}{\frac{x+6}{x-3}}$ — $\frac{x^2-4x+3}{x^2+11x+30}$

10. $\frac{\frac{12}{x+3}}{\frac{x^2+1}{x-2}}$ — $\frac{12x-24}{x^3+3x^2+x+3}$

Solve.

11. A messenger is required to deliver 10 packages per day. Each day, the messenger works only for as long as it takes to deliver the daily quota of 10 packages. On average, the messenger is able to deliver 2 packages per hour on Saturday and 4 packages per hour on Sunday. What is the messenger's average delivery rate on the weekend?

$2.\overline{66}$ packages per hour

LESSON 8-3

Practice C

Adding and Subtracting Rational Expressions

Add or subtract. Identify any x-values for which the expression is undefined.

1. $\frac{5x-1}{x+3} + \frac{3x}{2x+6}$ — $\frac{13x-2}{2x+6}$; $x \neq -3$

2. $\frac{7x}{3x^2} - \frac{2}{x+4}$ — $\frac{x^2+28x}{3x^2(x+4)}$; $x \neq -4$, $x \neq 0$

3. $\frac{x}{x-4} + \frac{x+1}{3x+1}$ — $\frac{4x^2-2x-4}{3x^2-11x-4}$; $x \neq -\frac{1}{3}$ and $x \neq 4$

4. $\frac{3}{x-5} - \frac{1}{x^2-7x+10}$ — $\frac{3x-7}{x^2-7x+10}$; $x \neq 5$, $x \neq 2$

5. $\frac{x}{4x-2} + \frac{3x+3}{4x+2}$ — $\frac{8x^2+4x-3}{8x^2-2}$; $x \neq \pm\frac{1}{2}$

6. $\frac{3x}{x^2-x-6} - \frac{5}{x^2-8x+15}$ — $\frac{3x^2-20x-10}{x^3-6x^2-x+30}$; $x \neq -2$, $x \neq 3$, $x \neq 5$

Simplify. Assume all expressions are defined.

7. $\frac{\frac{x+4}{x^2-8}}{\frac{x+4}{x-2}}$ — $\frac{x-2}{x^2-8}$

8. $\frac{\frac{x}{x+2}}{2x+\frac{x}{5}}$ — $\frac{5}{11x+22}$

9. $\frac{\frac{x-7}{x+2}}{\frac{x-5}{x+6}}$ — $\frac{x^2-x-42}{x^2-3x-10}$

10. $\frac{\frac{x-6}{x^2+3}}{\frac{x}{x^2+2x+1}}$ — $\frac{x^3-4x^2-11x-6}{x^3+3x}$

Solve.

11. The electric potential generated by a certain arrangement of electric charges is given by $\frac{e}{x-4} + \frac{e}{x+1}$, where e is the fundamental unit of electric charge and x measures the location where the potential is being measured. Express the electric potential as a rational expression.

$\frac{e(2x-3)}{x^2-3x-4}$

LESSON 8-3

Reteach

Adding and Subtracting Rational Expressions

Use a common denominator to add or subtract rational expressions.

Add: $\frac{6x+4}{x+5} + \frac{2x-8}{x+5}$.

Step 1 Add.

$\frac{6x+4}{x+5} + \frac{2x-8}{x+5} = \frac{6x+4+2x-8}{x+5}$ — The denominators are the same. Add the numerators.

$= \frac{6x+2x+4-8}{x+5}$ — Group like terms.

$= \frac{8x-4}{x+5}$ — Combine like terms.

Step 2 Identify x-values for which the expression is undefined.

$x \neq -5$ because -5 makes the denominator equal 0.

Subtract: $\frac{4x-3}{2x-1} - \frac{8x+2}{2x-1}$.

Step 1 Subtract.

$\frac{4x-3}{2x-1} - \frac{8x+2}{2x-1} = \frac{(4x-3)-(8x+2)}{2x-1}$ — The denominators are the same. Subtract the numerators.

$= \frac{4x-3-8x-2}{2x-1}$ — Use the Distributive Property.

$= \frac{-4x-5}{2x-1}$ — Combine like terms.

Step 2 Identify x-values for which the expression is undefined.

$x \neq \frac{1}{2}$ because $\frac{1}{2}$ makes the denominator equal 0.

Add or subtract.

1. $\frac{x-5}{x^2-4} + \frac{3x+2}{x^2-4}$ — $\frac{(x-5)+(3x+2)}{x^2-4}$ — $\frac{4x-3}{x^2-4}$; $x \neq$ −2, 2

2. $\frac{7x-5}{x+3} - \frac{4x-1}{x+3}$ — $\frac{(7x-5)-(4x-1)}{x+3}$ — $\frac{3x-4}{x+3}$; $x \neq$ −3

3. $\frac{2x-1}{x-1} - \frac{5x+4}{x-1}$ — $\frac{2x-1-(5x+4)}{(x-1)}$ — $\frac{-3x-5}{x-1}$; $x \neq$ 1

4. $\frac{4x+1}{3x+7} + \frac{9-x}{3x+7}$ — $\frac{3x+10}{3x+7}$; $x \neq -\frac{7}{3}$

5. $\frac{8-x}{x-3} - \frac{5-x}{x-3}$ — $\frac{3}{x-3}$; $x \neq$ 3

6. $\frac{5x+2}{x^2-1} - \frac{3x-7}{x^2-1}$ — $\frac{2x+9}{x^2-1}$; $x \neq$ ±1

LESSON 8-3

Reteach

Adding and Subtracting Rational Expressions (continued)

Use the least common denominator (LCD) to add rational expressions with different denominators. The process is the same as adding fractions with different denominators.

Add: $\frac{x-4}{x^2+2x-3} + \frac{2x}{x-1}$.

Step 1 Factor denominators completely.

$$\frac{x-4}{x^2+2x-3} + \frac{2x}{x-1} = \frac{x-4}{(x+3)(x-1)} + \frac{2x}{x-1}$$

Step 2 Find the LCD.
The LCD is the least common multiple of the denominators:
$(x+3)(x-1)$.

Step 3 Write each term of the expression using the LCD.

$$\frac{2x}{x-1} = \frac{2x}{x-1}\left(\frac{x+3}{x+3}\right) = \frac{2x^2+6x}{(x-1)(x+3)}$$

So, $\frac{x-4}{(x+3)(x-1)} + \frac{2x}{x-1} = \frac{x-4}{(x+3)(x-1)} + \frac{2x^2+6x}{(x-1)(x+3)}$

Step 4 Add the numerators and simplify.

$$\frac{x-4+2x^2+6x}{(x+3)(x-1)} = \frac{2x^2+7x-4}{(x+3)(x-1)}$$

Step 5 Identify x-values for which the expression is undefined.
$x \neq -3$ or 1 because both values make the denominator equal 0.

Add.

7. $\frac{x-1}{x^2-4} + \frac{3x}{x+2}$

$\frac{x-1}{(x+2)(x-2)} + \frac{3x}{x+2}$

$\frac{x-1}{(x+2)(x-2)} + \frac{3x}{x+2}\left(\frac{x-2}{x-2}\right)$

$\frac{x-1+(3x^2-6x)}{(x+2)(x-2)} = \frac{3x^2-5x-1}{(x+2)(x-2)}$

$x \neq$ $-2, 2$

8. $\frac{4x-1}{x^2+3x+2} + \frac{3}{x+1}$

$\frac{4x-1}{(x+2)(x+1)} + \frac{3}{x+1}\left(\frac{x+2}{x+2}\right)$

$\frac{4x-1+3x+6}{(x+2)(x+1)}$

$\frac{7x+5}{(x+2)(x+1)}$

$x \neq$ $-2, -1$

9. What is the LCD of $\frac{2x+1}{x^2-9}$ and $\frac{7}{x^2-x-6}$?

$(x-3)(x+3)(x+2)$

LESSON 8-3

Challenge

Partial Fractions

To add or subtract algebraic fractions, first find a common denominator.

$$\frac{3}{x+1} + \frac{2}{x-2} = \frac{3(x-2)+2(x+1)}{(x+1)(x-2)} = \frac{3x-6+2x+2}{(x+1)(x-2)} = \frac{5x-4}{x^2-x-2}$$

The decomposition of an algebraic fraction into partial fractions is the reverse of this process. The method shown in the example below can be used for fractions in which the degree of the numerator is less than the degree of the denominator, and in which the denominator can be factored into linear factors.

Example To decompose $\frac{5x-4}{x^2-x-2}$ into partial fractions:

Factor the denominator. $\frac{5x-4}{x^2-x-2} = \frac{5x-4}{(x+1)(x-2)}$

Express the fraction as the sum of two fractions, with the individual factors as denominators and the unknown numerators A and B. $\frac{5x-4}{(x+1)(x-2)} = \frac{A}{x+1} + \frac{B}{x-2}$

To clear fractions, multiply both sides of the rational equation by the LCD, $(x+1)(x-2)$. $5x-4 = A(x-2) + B(x+1)$

To determine A and B, first eliminate B by letting $x = -1$. Then eliminate A by letting $x = 2$.

$5(-1)-4 = A(-1-2)+B(-1+1)$ $\quad$ $5(2)-4 = A(2-2)+B(2+1)$

$-5-4 = -3A$ $\quad$ $10-4 = 3B$

$A = 3$ $\quad$ $B = 2$

Use the values for A and B to express the original fraction as the sum of two partial fractions. $\frac{5x-4}{(x+1)(x-2)} = \frac{3}{x+1} + \frac{2}{x-2}$

Express the given fraction as the sum of partial fractions.

1. $\frac{3x+18}{x^2+5x+4}$ $\quad \frac{5}{x+1} - \frac{2}{x+4}$

2. $\frac{6x^2+2x-4}{x^3-4x}$ $\quad \frac{1}{x} + \frac{2}{x+2} + \frac{3}{x-2}$

3. $\frac{7x^2+18x-19}{x^3+2x^2-5x-6}$ $\quad \frac{5}{x+1} + \frac{3}{x-2} - \frac{1}{x+3}$

The example above showed one way to find the numerators of the partial fractions. Another method can be used. After clearing fractions in the equation, regroup the terms on the right side to get an expression of the form $ax + b$. Then equate coefficients to obtain a system of two equations in A and B. Solve the system.

$5x-4 = A(x-2) + B(x+1)$
$5x-4 = (A+B)x + (-2A+B)$

$5 = A + B$ and $-4 = -2A + B$
$A = 3$ and $B - 2$

4. Consider a rational expression that has a repeated linear factor in its denominator, such as the one at right. Use the method of equating coefficients to find the partial fractions.

$$\frac{x^2-4x+6}{(x-1)^2(x-2)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x-2}$$

$\frac{-1}{x-1} - \frac{3}{(x-1)^2} + \frac{2}{x-2}$

LESSON 8-3

Problem Solving

Adding and Subtracting Rational Expressions

Vicki and Lorena motor downstream at about 6 knots (nautical miles per hour) in their boat. The return trip is against the current, and they can motor at only about 3 knots.

1. Vicki wants to find the average speed for the entire trip.

a. Write an expression for the time it takes to travel downstream plus the time it takes for the return trip if the distance in each direction is d. $\quad \frac{d}{6} + \frac{d}{3}$

b. What is the total distance they travel downstream and upstream in terms of d? $\quad 2d$

c. Write an expression for their average speed using the expressions for the total time and the total distance. $\quad \frac{2d}{\frac{d}{6} + \frac{d}{3}}$

d. Vicki says that the average speed is 4 knots. Lorena says that the average speed is 4.5 knots. Explain who is correct and why.

Vicki is correct. Possible answer: Lorena calculated the average speed as if it took the same amount of time for each leg of the trip. Vicki took into consideration the time for each leg.

2. If they delay the return trip until the current changes direction, they can motor back at 4 knots. What is the average speed for the entire trip under these conditions?

4.8 knots

Zak runs at an average speed of 7.0 miles per hour during the first half of a race and an average speed of 5.5 miles per hour during the second half of the race. Choose the letter for the best answer.

3. Which expression gives Zak's average speed for the entire race?

A $\frac{(7+5.5)}{2}$ $\quad$ B $\frac{12.5(7+5.5)d}{2}$

C $\frac{(38.5)d}{(7+5.5)}$ $\quad$ (D) $\frac{2(38.5)d}{(7+5.5)d}$

4. If Zak runs the race in 1.25 hours, what is the length of the race in miles?

A 3.85
B 6.25
(C) 7.7
D 12.5

5. In a later race, Zak increased his average speed during the second half of the race to 6.0 miles per hour. What is his average speed for this race in miles per hour?

A 6.42
(B) 6.46
C 6.52
D 6.56

6. It took Zak 1.6 hours to run this later race. What is the length of this race in miles?

A 5.17
B 7.28
C 9.55
(D) 10.34

LESSON 8-3

Reading Strategy

Compare and Contrast

The addition or subtraction of rational expressions can be compared to the addition or subtraction of fractions. As with fractions, in order to add or subtract rational expressions, the denominators must be the same. The least common multiple of the polynomials is the least common denominator (LCD).

	Add Fractions	Add Rational Expressions
	$\frac{3}{4} + \frac{1}{6}$	$\frac{5}{2x} + \frac{x+1}{x^2}$
Find the LCD.	12	$2x^2$
Rename each term using the LCD.	$\frac{9}{12} + \frac{2}{12}$	$\frac{5x}{2x^2} + \frac{2x+2}{2x^2}$
Add the numerators.	$\frac{11}{12}$	$\frac{7x+2}{2x^2}$

Find the least common multiple for each pair.

1. $2x^6$ and $6x$ $\quad 6x^6$
2. $10xy$ and $5x^4y^3$ $\quad 10x^4y^3$
3. $(x-8)(x+1)$ and $(x+1)$ $\quad (x-8)(x+1)$
4. $x^2 - 5x + 6$ and $x - 3$ $\quad (x-3)(x-2)$

A **complex fraction** is an expression that contains a fraction in the numerator, $\frac{\frac{8x}{x-3}}{x^2}$, the denominator, $\frac{3x+5}{\frac{13}{9x}}$, or both $\frac{\frac{4x}{x^2+1}}{\frac{x-3}{x+2}}$. To simplify, treat it as a division of rational expressions.

$$\frac{\frac{4(x-1)}{x^2-1}}{\frac{x-3}{x+1}} = \frac{4(x-1)}{x^2-1} \div \frac{x-3}{x+1} = \frac{4(x-1)}{(x+1)(x-1)} \cdot \frac{x+1}{x-3} = \frac{4}{x-3}$$

Write the division expression and the corresponding multiplication expression you could use to simplify the complex fraction.

5. $\frac{\frac{8x}{x-3}}{\frac{x^2}{2}}$ $\quad \frac{8x}{x-3} \div \frac{x^2}{2};\ \frac{8x}{x-3} \cdot \frac{2}{x^2}$

6. $\frac{\frac{2}{x-1}}{\frac{x+1}{x^3}}$ $\quad \frac{2}{x-1} \div \frac{x+1}{x^3};\ \frac{2}{x-1} \cdot \frac{x^3}{x+1}$