Name _____ Date _____ Class _____

LESSON 8-2

Reteach

Multiplying and Dividing Rational Expressions

Examples of rational expressions: $\frac{3}{x}$, $\frac{x+1}{x+2}$, and $\frac{x+3}{2x^2}$

Undefined at $x = 0$

Undefined at $x = -2$

Undefined at $x = 0$

When simplifying a **rational expression**:

- Factor the numerator and the denominator completely.
- Divide out any common factors.
- Identify any x-values for which the expression is undefined.

Simplify: $\frac{24x^6}{8x^2}$.

$x \neq 0$, because $8x^2$ is undefined at $x = 0$.

$$\frac{24x^6}{8x^2} = \frac{8 \cdot 3}{8} x^{6-2} = 3x^4$$

Use the Quotient of Powers Property.

Simplify: $\frac{x^2 - 2x - 8}{x^2 + x - 2}$.

First, factor the numerator and the denominator.

$$\frac{x^2 - 2x - 8}{x^2 + x - 2} = \frac{(x-4)(x+2)}{(x+2)(x-1)} = \frac{(x-4)(x+2)}{(x+2)(x-1)} = \frac{(x-4)}{(x-1)} = \frac{x-4}{x-1}$$

$x \neq -2$ and $x \neq 1$

Divide out common factors.

Simplify.

1. $\frac{x^2 - 2x - 3}{x^2 + 6x + 5}$

$\frac{(x+1)(x-3)}{(x+1)(x+5)}$

_____

$x \neq$ _____

2. $\frac{20x^9}{4x^3}$

_____

_____

$x \neq$ _____

3. $\frac{x^2 - 4x}{x^2 - 5x + 4}$

_____

_____

$x \neq$ _____

Name _____ Date _____ Class _____

LESSON 8-2

Reteach

Multiplying and Dividing Rational Expressions (continued)

Multiplying rational expressions is similar to multiplying fractions.

Multiply: $\frac{15x^2y^3}{4x^3y^5} \cdot \frac{2x^4y^3}{3xy^2}$.

$$\frac{15x^2y^3}{4x^3y^5} \cdot \frac{2x^4y^3}{3xy^2} = \frac{15}{4} \cdot \frac{2}{3} \cdot \frac{x^2x^4}{x^3x} \cdot \frac{y^3y^3}{y^5y^2}$$

Group like factors.

$$= \frac{5}{2} \cdot \frac{x^6}{x^4} \cdot \frac{y^6}{y^7}$$

Simplify constants. Add exponents to multiply.

$$= \frac{5}{2} \cdot x^2 \cdot \frac{1}{y}$$

Subtract exponents to divide.

$$= \frac{5x^2}{2y}$$

Simplify.

Multiplying rational expressions is similar to simplifying rational expressions.

Multiply: $\frac{x+3}{6x-6} \cdot \frac{x-1}{x^2-9}$.

$$\frac{x+3}{6x-6} \cdot \frac{x-1}{x^2-9} = \frac{x+3}{6(x-1)} \cdot \frac{x-1}{(x+3)(x-3)}$$

Completely factor all numerators and denominators.

$$= \frac{\cancel{x+3}}{6(\cancel{x-1})} \cdot \frac{\cancel{x-1}}{(\cancel{x+3})(x-3)}$$

Divide out common factors.

$$= \frac{1}{6(x-3)}$$

Simplify.

To divide rational expressions, multiply by the reciprocal.

$$\frac{x+7}{x-2} \div \frac{x^2-49}{2x-4} = \frac{x+7}{x-2} \cdot \frac{2x-4}{x^2-49} = \frac{\cancel{x+7}}{\cancel{x-2}} \cdot \frac{2(x-2)}{(x-7)(x+7)} = \frac{2}{x-7}$$

Multiply. Assume that all expressions are defined.

4. $\frac{12x^5y^2}{6x^2y^4} \cdot \frac{9x^3y}{3x^2y^3}$

5. $\frac{2x-2}{x+4} \cdot \frac{x^2+4x}{x^2-3x+2}$

6. $\frac{8x+16}{x^2-1} \cdot \frac{x+1}{4x+8}$

7. $\frac{3x^3y}{5xy^2} \div \frac{9xy^3}{15y}$

8. $\frac{4x-8}{x^2-4} \div \frac{3x}{x+2}$

9. $\frac{x^2+2x-3}{x^2-9} \div \frac{x^2+3x-4}{x^2-2x-3}$

LESSON 8-2

Practice A

Multiplying and Dividing Rational Expressions

Answer each question.

1. What could make a rational expression undefined?

   Possible answer: If the denominator is 0, then the expression is undefined because division by 0 is impossible.

2. What value of x gives a denominator equal to 0 for the expression $\frac{x+4}{x-3}$?

   $x = 3$

Simplify. Identify any x-values for which the expression is undefined.

3. $\frac{5x}{x^3}$

   $\frac{5}{x^2}; x \neq 0$

4. $\frac{x^2+3x}{x^3}$

   $\frac{x+3}{x^2}; x \neq 0$

5. $\frac{4x+12}{6x+18}$

   $\frac{2}{3}; x \neq -3$

6. $\frac{2x+7}{x+3}$

   $\frac{2x+7}{x+3}; x \neq -3$

Complete. Assume that all expressions are defined.

7. $\frac{3x}{2y} \cdot \frac{2}{y} = \frac{3x}{y^2}$

8. $\frac{2x^5}{9y^4} \cdot \frac{3y^2}{x} = \frac{2x^4}{3y^2}$

9. $\frac{x+2}{x-1} \cdot \frac{4x-4}{(x+2)(x-2)} = \frac{4}{x-2}$

10. $\frac{x^3+8x^2}{x} \cdot \frac{x}{x^3+4x^2} = \frac{x+8}{x+4}$

11. $\frac{2x}{3y} \div \frac{2x^2}{3y^2} = \frac{2x}{3y} \cdot \frac{3y^2}{2x^2} = \frac{y}{x}$

12. $\frac{x+1}{x-2} \div \frac{5(x+1)}{2(x+2)} = \frac{x+1}{x-2} \cdot \frac{2(x+2)}{5(x+1)} = \frac{2x+4}{5x-10}$

13. $\frac{2y}{x+4} \div \frac{4y^2}{x+4} = \frac{2y}{x+4} \cdot \frac{x+4}{4y^2} = \frac{1}{2y}$

Solve.

14. $\frac{2x^2+2x}{x+1} = 8$

    $x = 4$

15. $\frac{9x+9}{x^3+x^2} = 1$

    $x = 3$

16. $\frac{5x^2+15x}{x+3} = 10$

    $x = 2$

17. $\frac{x^2+3x+2}{2x^2+4} = 1$

    $x = 1$

LESSON 8-2

Practice B

Multiplying and Dividing Rational Expressions

Simplify. Identify any x-values for which the expression is undefined.

1. $\frac{x^2+3x+2}{x^2-3x-4}$

   $\frac{x+2}{x-4}; x \neq -1, x \neq 4$

2. $\frac{4x^6}{2x^4}$

   $2x^2; x \neq 0$

3. $\frac{x^2-x^3}{2x^2-5x+3}$

   $\frac{-x^2}{2x-3}; x \neq 1, x \neq \frac{3}{2}$

4. $\frac{x^3+x^2-20x}{x^2-16}$

   $\frac{x^2+5x}{x+4}; x \neq 4, x \neq -4$

5. $\frac{3x^2-9x-12}{6x^2+9x+3}$

   $\frac{x-4}{2x+1}; x \neq -1, x \neq -\frac{1}{2}$

6. $\frac{9-3x}{15-2x-x^2}$

   $\frac{3}{x+5}; x \neq 3, x \neq -5$

Multiply. Assume all expressions are defined.

7. $\frac{4x+16}{2x+6} \cdot \frac{x^2+2x-3}{x+4}$

   $2x-2$

8. $\frac{x+3}{x-1} \cdot \frac{x^2-2x+1}{x^2+5x+6}$

   $\frac{x-1}{x+2}$

Divide. Assume all expressions are defined.

9. $\frac{5x^6}{x^2y} \div \frac{10x^2}{y}$

   $\frac{x^2}{2}$

10. $\frac{x^2-2x-8}{x^2-2x-15} \div \frac{2x^2-8x}{2x^2-10x}$

    $\frac{x+2}{x+3}$

Solve. Check your solution.

11. $\frac{x^2+x-12}{x-3} = 15$

    $x = 11$

12. $\frac{2x^2+8x-10}{2x^2+14x+20} = 4$

    $x = -3$

Solve.

13. The distance, d, traveled by a car undergoing constant acceleration, a, for a time, t, is given by $d = v_0 t + \frac{1}{2}at^2$, where v_0 is the initial velocity of the car. Two cars are side by side with the same initial velocity. One car accelerates, $a = A$, and the other car does not accelerate, $a = 0$. Write an expression for the ratio of the distance traveled by the accelerating car to the distance traveled by the nonaccelerating car as a function of time.

    $1 + \frac{At}{2v_0}$

LESSON 8-2

Practice C

Multiplying and Dividing Rational Expressions

Multiply. Assume all expressions are defined.

1. $\frac{2x^3y^6}{5x^2y^3} \cdot \frac{15x^2y^2}{10x^4y}$

   $\frac{3y^4}{5x}$

2. $\frac{2x+14}{x^2-25} \cdot \frac{8x+40}{6x+42}$

   $\frac{8}{3(x-5)}$

3. $\frac{4x^3+12x^2}{2x^3-16x^2} \cdot \frac{2x^2-10x-48}{x+3}$

   $4(x+3)$

4. $\frac{3x^2+15x-18}{36x^3-12x^2} \cdot \frac{9x^3-3x^2}{9x^2+36x-108}$

   $\frac{x-1}{12(x-2)}$

Divide. Assume all expressions are defined.

5. $\frac{4x^3y^2}{2x^6y} \div \frac{xy+2y}{x^2-3x-10}$

   $\frac{2(x-5)}{x^3}$

6. $\frac{3x^2+6x-24}{x^2-x-20} \div \frac{3x^3-9x^2+6x}{x}$

   $\frac{1}{(x-5)(x-1)}$

7. $\frac{4x^3y^{12}}{y^5x^2} \div \frac{6y^7m^3x^3}{3m^3y}$

   $\frac{2y}{x^2}$

8. $\frac{4x^2-12x-72}{8x^2+32x-40} \div \frac{x^2-9x+18}{x^2+2x-15}$

   $\frac{x+3}{2(x-1)}$

Solve. Check your solution.

9. $\frac{4x^2-4x-168}{2x+12} = 20$

   $x = 17$

10. $\frac{x^2-3x-18}{x+3} = 13$

    $x = 19$

11. $\frac{x^2+3x-10}{x-2} = 7$

    No solution

12. $\frac{2x^2-18x+40}{2x-10} = 5$

    $x = 9$

Solve.

13. The formula for the volume of a cylinder is $\pi r^2 h$ and the formula for its surface area is $2\pi r^2 + 2\pi rh$, where r is the radius and h is the height. A cylindrical industrial storage tank has a surface area to-volume ratio of 3. If the height of the cylindrical tank is 2 meters, what is the radius?

    1 meter

LESSON 8-2

Reteach

Multiplying and Dividing Rational Expressions

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Simplify: $\frac{24x^6}{8x^2}$.

$x \neq 0$, because $8x^2$ is undefined at $x = 0$.

$\frac{24x^6}{8x^2} = \frac{8 \cdot 3}{8} x^{6-2} = 3x^4$

Use the Quotient of Powers Property.

Simplify: $\frac{x^2-2x-8}{x^2+x-2}$.

First, factor the numerator and the denominator.

$\frac{x^2-2x-8}{x^2+x-2} = \frac{(x-4)(x+2)}{(x+2)(x-1)} = \frac{(x-4)(x+2)}{(x+2)(x-1)} = \frac{(x-4)}{(x-1)} = \frac{x-4}{x-1}$

$x \neq -2$ and $x \neq 1$

Divide out common factors.

Simplify.

1. $\frac{x^2-2x-3}{x^2+6x+5}$

   $\frac{(x+1)(x-3)}{(x+1)(x+5)}$

   $\frac{x-3}{x+5}$,

   $x \neq$ $-1, -5$

2. $\frac{20x^9}{4x^3}$

   $\frac{20}{4} \cdot \frac{x^9}{x^3}$

   $5x^6$

   $x \neq$ 0

3. $\frac{x^2-4x}{x^2-5x+4}$

   $\frac{x(x-4)}{(x-4)(x-1)}$

   $\frac{x}{x-1}$

   $x \neq$ $1, 4$

LESSON 8-2

Reteach

Multiplying and Dividing Rational Expressions (continued)

Multiplying rational expressions is similar to multiplying fractions.

Multiply: $\frac{15x^2y^3}{4x^3y^5} \cdot \frac{2x^4y^3}{3xy^2}$.

$$\frac{15x^2y^3}{4x^3y^5} \cdot \frac{2x^4y^3}{3xy^2} = \frac{15}{4} \cdot \frac{2}{3} \cdot \frac{x^2x^4}{x^3x} \cdot \frac{y^3y^3}{y^5y^2}$$

Group like factors.

$$= \frac{5}{2} \cdot \frac{x^6}{x^4} \cdot \frac{y^6}{y^7}$$

Simplify constants. Add exponents to multiply.

$$= \frac{5}{2} \cdot x^2 \cdot \frac{1}{y}$$

Subtract exponents to divide.

$$= \frac{5x^2}{2y}$$

Simplify.

Multiplying rational expressions is similar to simplifying rational expressions.

Multiply: $\frac{x+3}{6x-6} \cdot \frac{x-1}{x^2-9}$.

$$\frac{x+3}{6x-6} \cdot \frac{x-1}{x^2-9} = \frac{x+3}{6(x-1)} \cdot \frac{x-1}{(x+3)(x-3)}$$

Completely factor all numerators and denominators.

$$= \frac{\cancel{x+3}}{6(\cancel{x-1})} \cdot \frac{\cancel{x-1}}{(\cancel{x+3})(x-3)}$$

Divide out common factors.

$$= \frac{1}{6(x-3)}$$

Simplify.

To divide rational expressions, multiply by the reciprocal.

$$\frac{x+7}{x-2} \div \frac{x^2-49}{2x-4} = \frac{x+7}{x-2} \cdot \frac{2x-4}{x^2-49} = \frac{\cancel{x+7}}{\cancel{x-2}} \cdot \frac{2(x-2)}{(x-7)(x+7)} = \frac{2}{x-7}$$

Multiply. Assume that all expressions are defined.

4. $\frac{12x^5y^2}{6x^2y^4} \cdot \frac{9x^3y}{3x^2y^3}$ — $\frac{6x^4}{y^4}$

5. $\frac{2x-2}{x+4} \cdot \frac{x^2+4x}{x^2-3x+2}$ — $\frac{2x}{x-2}$

6. $\frac{8x+16}{x^2-1} \cdot \frac{x+1}{4x+8}$ — $\frac{2}{x-1}$

7. $\frac{3x^3y}{5xy^2} \div \frac{9xy^3}{15y}$ — $\frac{x}{y^3}$

8. $\frac{4x-8}{x^2-4} \div \frac{3x}{x+2}$ — $\frac{4}{3x}$

9. $\frac{x^2+2x-3}{x^2-9} \div \frac{x^2+3x-4}{x^2-2x-3}$ — $\frac{x+1}{x+4}$

LESSON 8-2

Challenge

The Skill Factor

The process of multiplying and dividing rational expressions requires an understanding of the process of factoring. The following exercises will challenge some of your factoring skills. Remember to follow all factoring rules. Assume all expressions are defined.

1. Simplify the rational expression. $\frac{x^{3y} + 4x^{2y} - 3x^y - 12}{x^y + 4}$ — $x^{2y} - 3$

2. Simplify the rational expression. $\frac{5x^{3a} + 5x^{2a}y^a}{x^{2a} - y^{2a}}$ — $\frac{5x^{2a}}{x^a - y^a}$

3. Simplify the rational expression. $\frac{j^3k + j^3 - k^4 - k^3}{j^2k + j^2 + jk^2 + jk + k^3 + k^2}$ — $j - k$

4. Multiply. $\frac{4r^2 + 2rs + s^2}{2r + s} \cdot \frac{4r^2 - s^2}{8r^3 - s^3}$ — 1

5. Multiply. $\frac{2a^2 - 2a - 12}{a^2 - 49} \cdot \frac{4a^2 - 1}{2a^2 + 5a + 2} \cdot \frac{2a^2 - 13a - 7}{2a^2 - 7a + 3}$ — $\frac{2(2a+1)}{a+7}$

6. Multiply. $\frac{p^3 - 4p^2 + p - 4}{2p^3 - 8p^2 + p - 4} \cdot \frac{2p^3 + 2p^2 + p + 1}{p^4 - p^3 + p^2 - p}$ — $\frac{p+1}{p(p-1)}$

7. Divide. $\frac{\frac{m^3 + n^3}{mp - mq - np + nq}}{\frac{mn - m^2 - n^2}{mp - mq + np - nq}}$ — $\frac{(m+n)^2}{n-m}$

8. Divide. $\frac{\frac{a^{2n} - 1}{a^{2n} + 3a^n + 2}}{\frac{a^{2n} + a^n - 12}{a^{2n} - a^n - 6}}$ — $\frac{a^n - 1}{a^n + 4}$

9. Divide. $\frac{\left(\frac{x-y}{4z}\right)}{\left(\frac{y-x}{z}\right)\left(\frac{x-y}{z^2}\right)}$ — $\frac{z^2}{4(y-x)}$

10. Solve the following equation for y in terms of x, and write the resulting expression for y in simplest form. Identify any excluded values of x.
$x^2(y+1) = 9(y+1) + 4x + 12$

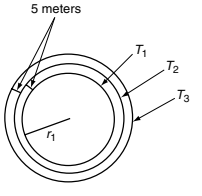
$y = \frac{7-x}{x-3}$; $x \neq -3$ or 3

LESSON 8-2

Problem Solving

Multiplying and Dividing Rational Expressions

Anders designs a running field that consists of three concentric tracks as shown in the diagram.



1. How do the lengths of each track compare?

   a. Write an equation for the length of the inner track, T_1, in terms of radius, r_1.
   $T_1 = 2\pi r_1$

   b. Write an equation for the length of the middle track, T_2, in terms of radius r_1.
   $T_2 = 2\pi(r_1 + 5)$

   c. Then write a rational expression for the ratio of the length of track T_2 to the length of track T_1 in terms of radius r_1.
   $\frac{r_1 + 5}{r_1}$

2. Mari writes the expression $\frac{(r_1 + 10)(r_1 - 5)}{r_1^2 - 25}$ for the ratio of the length of the outer track, T_3, to that of the middle track, T_2. Anders thinks that is the wrong expression. Simplify Mari's expression to determine if she is correct. Explain.
   $\frac{(r_1 + 10)(r_1 - 5)}{(r_1^2 - 25)} = \frac{(r_1 + 10)(r_1 - 5)}{(r_1 + 5)(r_1 - 5)} = \frac{r_1 + 10}{r_1 + 5}$;
   Mari is correct.

3. Anders sets the radius of the inner track, T_1, at 70 meters.

   a. How many times longer is the middle track, T_2, than the inner track, T_1? $\frac{T_2}{T_1} = 1.071$

   b. How many times longer is the outer track, T_3, than the middle track, T_2? $\frac{T_3}{T_2} = 1.067$

   c. How many times longer is the outer track, T_3, than the inner track, T_1? $\frac{T_3}{T_1} = 1.143$

Choose the letter for the best answer.

4. How many times as large is the area enclosed by the outer track, T_3, than the area enclosed by the inner track, T_1?
   A $\left(\frac{10}{r_1}\right)$ B $\left(\frac{10}{r_1}\right)^2$
   C $\left(\frac{r_1 + 10}{r_1}\right)$ D $\left(\frac{r_1 + 10}{r_1}\right)^2$

5. What is the ratio of the area between the inner track and the outer track to the area enclosed by the inner track?
   A $20\left(\frac{r_1 + 5}{r_1^2}\right)$ B $\frac{(r_1 + 10)^2 - 1}{r_1^2}$
   C $\pi\left(\frac{r_1 + 10}{r_1}\right)^2$ D $\pi\left(\frac{10}{r_1}\right)^2$

LESSON 8-2

Reading Strategy

Follow a Procedure

Dividing rational expressions can be thought of as multiplying by the reciprocal of the divisor. In order to divide a rational expression, rewrite the division as multiplication, simplify, and multiply.

STEP 1 **STEP 2** **STEP 3**

$\frac{5x^2}{6y^2} \div \frac{10x}{3y} \rightarrow \frac{5x^2}{6y^2} \cdot \frac{3y}{10x} \rightarrow \frac{x}{2y} \cdot \frac{1}{2} \rightarrow \frac{x}{4y}$

Multiply by the reciprocal of the divisor.

Simplify by dividing out common factors.

Multiply.

Answer each question.

1. For $\frac{8x}{8x - 16}$,
   a. Simplify the expression. $\frac{x}{x-2}$
   b. for which value of x is the expression undefined? $x = 2$
   c. Explain why the expression is undefined for this value.
   Because $x = 2$ makes the denominator of the expression equal to 0

2. For the expression $\frac{6x^3y^2}{7z^4} \div \frac{2xy^2}{21z^2}$
   a. Rewrite as multiplication. $\frac{6x^3y^2}{7z^4} \cdot \frac{21z^2}{2xy^2}$
   b. Simplify. $\frac{3x^2}{z^2} \cdot \frac{3}{1}$
   c. Multiply. $\frac{9x^2}{z^2}$
   d. For which value is the resulting expression undefined? $z = 0$

3. For the expression $\frac{3(x-1)}{2(x+2)} \div \frac{9(x-1)}{4(x+2)}$
   a. Rewrite as multiplication. $\frac{3(x-1)}{2(x+2)} \cdot \frac{4(x+2)}{9(x-1)}$
   b. Simplify. $\frac{1}{1} \cdot \frac{2}{3}$
   c. Multiply. $\frac{2}{3}$
   d. For which value is the resulting expression undefined? The resulting expression is never undefined.

4. Explain how to check the results of division.
   By multiplying the result by the divisor; if it is correct their product should be the dividend.