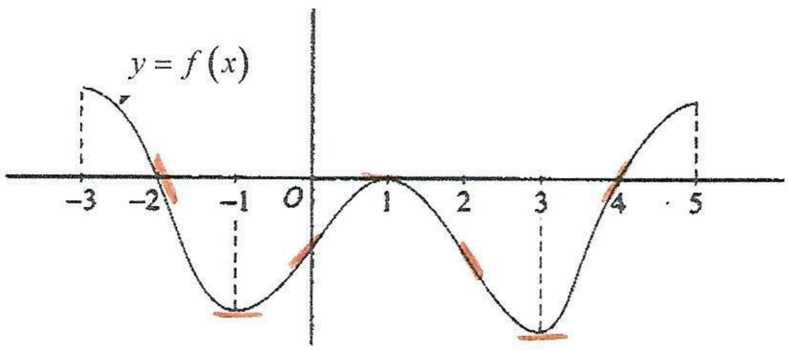
AP Calculus BC Name Key Period

Unit 1 – Wkst 5 – Derivatives from Graphs and Tables



1. Using the graph above, answer the following:
   What is the sign of $f'(-2)$, $f'(-1)$, $f'(0)$, $f'(1)$, $f'(2)$, $f'(3)$, and $f'(4)$
   $-$ $\quad 0 \quad +$ $\quad 0 \quad -$ $\quad 0 \quad +$

2. Suppose f and g are differentiable functions with the values shown in the table below. For each of the following functions h, find $h'(2)$.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
2	3	4	5	-2

a) $h(x) = f(x) + g(x)$

$h'(x) = f'(x) + g'(x)$

$h'(2) = f'(2) + g'(2)$

$= 5 + (-2)$

$= 3$

b) $h(x) = f(x)g(x)$

$h'(x) = f'(x)g(x) + g'(x)f(x)$

$h'(2) = f'(2)g(2) + g'(2)f(2)$

$= (5)(4) + (-2)(3)$

$= 20 - 6$

$= 14$

c) $h(x) = \dfrac{f(x)}{g(x)}$

$h'(x) = \dfrac{f'(x)g(x) - g'(x)f(x)}{(g(x))^2}$

$h'(2) = \dfrac{(5)(4) - (-2)(3)}{4^2}$

$= \dfrac{20+6}{16} = \dfrac{26}{16} = \dfrac{13}{8}$

3. Suppose f and g are differentiable functions with the values shown in the table below. For each of the following functions h, find $h'(2)$.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
2	5	5	e	$\sqrt{2}$
5	2	8	π	7

a) $h(x) = f(g(x))$

$h'(x) = f'(g(x)) \cdot g'(x)$

$h'(2) = f'(5) \cdot \sqrt{2}$

$= \pi\sqrt{2}$

b) $h(x) = g(f(x))$

$h'(x) = g'(f(x)) \cdot f'(x)$

$h'(2) = g'(5) \cdot e$

$= 7e$

c) $h(x) = f(f(x))$

$h'(x) = f'(f(x)) \cdot f'(x)$

$h'(2) = f'(5) \cdot e$

$= \pi e$

x	1	2	3	4
$f(x)$	3	2	1	4
$g(x)$	1	4	2	3
$f'(x)$	2	1	4	3
$g'(x)$	4	2	3	1

4. Suppose we are given the data in the table for the differentiable functions f and g and their derivatives.

a) Find $h(4)$ if $h(x)=f(g(x))$

$$h(4)=f(g(4)) = f(3) = 1$$

b) Find $h'(4)$ if $h(x)=f(g(x))$

$$h'(x)=f'(g(x))\cdot g'(x)$$
$$h'(4)=f'(3)\cdot 1 = (4)(1) = 4$$

c) Find $h(4)$ if $h(x)=g(f(x))$

$$h(4)=g(f(4)) = g(4) = 3$$

d) Find $h'(4)$ if $h(x)=g(f(x))$

$$h'(x)=g'(f(x))\cdot f'(x)$$
$$h'(4)=g'(4)\cdot 3 = (1)(3) = 3$$

e) Find $h'(4)$ if $h(x)=\dfrac{f(x)}{g(x)}$

$$h'(x)=\frac{f'(x)g(x)-g'(x)f(x)}{(g(x))^2}$$
$$h'(4)=\frac{(3)(3)-(1)(4)}{3^2} = \frac{9-4}{9} = \frac{5}{9}$$

f) Find $h'(4)$ if $h(x)=f(x)g(x)$

$$h'(x)=f'(x)g(x)+g'(x)f(x)$$
$$h'(4)=(3)(3)+(1)(4) = 9+4 = 13$$

5. Using the information in #4, explain why there must be at least one value of x in the interval $(1,4)$ such that $g(x)=e$.

- $\left.\begin{matrix} g(3)=2 \\ g(4)=3 \end{matrix}\right\}$ $g(3)<e<g(4)$
- g is given to be differentiable $\Rightarrow$ g is continuous
- By IVT, $\exists\, x$ in $3<x<4 \ni g(x)=e$

6. Using the information in #4, explain why there must be at least one value of x in the interval $(1,4)$ such that $f(x)=\pi$.

- $\left.\begin{matrix} f(1)=3 \\ f(4)=4 \end{matrix}\right\}$ $f(1)<\pi<f(4)$
- f is given to be differentiable $\Rightarrow$ f is continuous
- By IVT, $\exists\, x$ in $1<x<4 \ni f(x)=\pi$