AP Calculus BC Name: Key Period

Unit 1 – Wkst 4 – Product Rule, Quotient Rule, Chain Rule

No Calculator.

Find each derivative using proper notation.

1. $y = 25$

$y' = 0$

2. $h(x) = 6\sqrt{x} + 3\sqrt[3]{x} = 6x^{\frac{1}{2}} + 3x^{\frac{1}{3}}$

$h'(x) = \boxed{3x^{-\frac{1}{2}} + x^{-2/3}}$

$= \dfrac{3}{\sqrt{x}} + \dfrac{1}{x^{2/3}}$

3. $f(\theta) = 4\theta - 5\sin\theta$

$f'(\theta) = 4 - 5\cos\theta$

4. Find the equation of the line tangent to the graph of $f(x) = \dfrac{27}{x^3}$ at the point $(3,1)$.

Point: $(3,1)$

Slope: $f(x) = 27x^{-3}$

$f'(x) = -81x^{-4}$

$f'(3) = -81\left(\frac{1}{3^4}\right) = -1$

EQ of Tang Line:

$y - 1 = -1(x-3)$

Find each derivative using proper notation.

5. $f(x) = (5x^2+8)(x^2-3x-6)$

$f'(x) = (10x)(x^2-3x-6) + (5x^2+8)(2x-3)$

6. $h(x) = \sqrt{x}\cos x$

$h'(x) = \boxed{\left(\tfrac{1}{2}x^{-\frac{1}{2}}\right)(\cos x) + (\sqrt{x})(-\sin x)}$

$= \dfrac{\cos x}{2\sqrt{x}} - \sqrt{x}\sin x$

7. $f(x) = \dfrac{x^2+x-1}{x^2-2}$

$f'(x) = \dfrac{(2x+1)(x^2-2) - (x^2+x-1)(2x)}{(x^2-2)^2}$

8. $y = \dfrac{x^4}{\cos x}$

$y' = \dfrac{4x^3\cos x + x^4\sin x}{\cos^2 x}$

9. $y = 4x^2\sec x$

$y' = 8x\sec x + 4x^2\sec x\tan x$

10. $g(x) = \sin x\cos x$

$g'(x) = \boxed{\cos x\cos x - \sin x\sin x}$

$= \cos^2 x - \sin^2 x$

$= \cos 2x$

Find the second derivative of the function. Use proper notation always!

11. $g(t) = -8t^3 - 3t + 12$

$g'(t) = -24t^2 - 3$

$g''(t) = -48t$

12. $f(\theta) = 3\tan\theta$

$f'(\theta) = 3\sec^2\theta = 3(\sec\theta)^2$

$f''(\theta) = 6(\sec\theta)^1(\sec\theta\tan\theta)$

$= 6\sec^2\theta\tan\theta$

Find each derivative.

13. $y=(7x+3)^4$

$y'=\boxed{4(7x+3)^3(7)}$

$=28(7x+3)^3$

14. $y=\dfrac{1}{x^2+4}=(x^2+4)^{-1}$

$y'=\boxed{-1(x^2+4)^{-2}(2x)}$

$=\dfrac{-2x}{(x^2+4)^2}$

15. $y=5\cos(9x+1)$

$y'=-5\sin(9x+1)\cdot 9$

$=-45\sin(9x+1)$

16. $y=\dfrac{x}{2}-\dfrac{\sin 2x}{4}$

$y=\frac{1}{2}x-\frac{1}{4}\sin 2x$

$y'=\frac{1}{2}-\frac{1}{4}(\cos 2x)2$

$=\frac{1}{2}-\frac{1}{2}\cos 2x$

17. $y=x(6x+1)^5$

$y'=\boxed{(1)(6x+1)^5+(x)(5)(6x+1)^4(6)}$

$=(6x+1)^5+30x(6x+1)^4$

$=(6x+1)^4(6x+1+30x)$

$=(6x+1)^4(36x+1)$

MC answer!

18. $f(x)=\dfrac{3x}{\sqrt{x^2+1}}$

$f'(x)=\boxed{\dfrac{3\sqrt{x^2+1}-3x\left[\frac{1}{2}(x^2+1)^{-\frac{1}{2}}(2x)\right]}{(\sqrt{x^2+1})^2}}$

$=\dfrac{3\sqrt{x^2+1}-\dfrac{3x^2}{\sqrt{x^2+1}}}{x^2+1}\cdot\dfrac{\sqrt{x^2+1}}{\sqrt{x^2+1}}$

$=\dfrac{3(x^2+1)-3x^2}{(x^2+1)^{3/2}}=\dfrac{3}{(x^2+1)^{3/2}}$

MC answer!

19. $h(x)=5\tan^3(4x^2-3)=5[\tan(4x^2-3)]^3$

$h'(x)=15[\tan(4x^2-3)]^2\sec^2(4x^2-3)\cdot 8x$

$=120x\tan^2(4x^2-3)\sec^2(4x^2-3)$

20. $g(x)=\dfrac{2\sin^2(3x)}{x}=\dfrac{2[\sin(3x)]^2}{x}$

$g'(x)=\dfrac{4\sin(3x)\cos(3x)(3)(x)-(1)2\sin^2(3x)}{x^2}$

$=\dfrac{12x\sin(3x)\cos(3x)-2\sin^2(3x)}{x^2}$

OR $\dfrac{2\sin(3x)[6x\cos(3x)-\sin(3x)]}{x^2}$

21. Find the value(s) of x in the interval $0\le x\le 2\pi$ at which the slopes of $y=\cos^2 x$ and $y=\sin^2 x$ are equal.

$y=\cos^2 x$ | $y=\sin^2 x$

$y'=2\cos x(-\sin x)$ | $y'=2\sin x\cos x$

$-2\cos x\sin x \overset{\text{set}}{=} 2\sin x\cos x$

$0=4\sin x\cos x$

$\sin x=0$ at $x=0,\pi,2\pi$

$\cos x=0$ at $x=\frac{\pi}{2},\frac{3\pi}{2}$

So the slopes are equal at $x=0,\frac{\pi}{2},\pi,\frac{3\pi}{2},2\pi$