AP Calculus BC Name: Key Period

Unit 1– Wkst 3 – Definitions of Derivative, Differentiability, Power Rule for Derivatives

No calculator.

Find the derivative using the definition of derivative. Show all work. Use proper notation.

1. $f(x)=2x^2+x-1$

$$f'(x)=\lim_{h\to 0}\frac{[2(x+h)^2+(x+h)-1]-[2x^2+x-1]}{h}$$
$$=\lim_{h\to 0}\frac{2x^2+4xh+2h^2+x+h-1-2x^2-x+1}{h}$$
$$=\lim_{h\to 0}\frac{4xh+2h^2+h}{h}$$
$$=\lim_{h\to 0} 4x+2h+1$$
$$=4x+1$$

Definition of Derivative of $f(x)$: $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

2. $f(x)=\frac{1}{x-1}$

$$f'(x)=\lim_{\Delta x\to 0}\frac{\frac{1}{x+\Delta x-1}-\frac{1}{x-1}}{\Delta x}$$ Mult by common denom.
$$=\lim_{\Delta x\to 0}\frac{(x-1)-(x+\Delta x-1)}{\Delta x(x+\Delta x-1)(x-1)}$$
$$=\lim_{\Delta x\to 0}\frac{x-1-x-\Delta x+1}{\Delta x(x+\Delta x-1)(x-1)}$$
$$=\lim_{\Delta x\to 0}\frac{-\Delta x}{\Delta x(x+\Delta x-1)(x-1)}$$
$$=\lim_{\Delta x\to 0}\frac{-1}{(x+\Delta x-1)(x-1)}$$
$$=-\frac{1}{(x-1)^2}$$

3. $g(x)=\sqrt{x-4}$

$$g'(x)=\lim_{h\to 0}\frac{\sqrt{x+h-4}-\sqrt{x-4}}{h}$$ mult. by conjugate
$$=\lim_{h\to 0}\frac{(x+h-4)-(x-4)}{h(\sqrt{x+h-4}+\sqrt{x-4})}$$
$$=\lim_{h\to 0}\frac{x+h-4-x+4}{h(\sqrt{x+h-4}+\sqrt{x-4})}$$
$$=\lim_{h\to 0}\frac{h}{h(\sqrt{x+h-4}+\sqrt{x-4})}$$
$$=\lim_{h\to 0}\frac{1}{\sqrt{x+h-4}+\sqrt{x-4}}$$
$$=\frac{1}{2\sqrt{x-4}}$$

Find the derivative of each function at the given value of x, using the alternative form of the definition of derivative. Show all work. Use proper notation.

5. $f(x)=x^2-4$ at $x=1$

$$f'(1)=\lim_{x\to 1}\frac{(x^2-4)-(-3)}{x-1}$$
$$=\lim_{x\to 1}\frac{x^2-1}{x-1}$$
$$=\lim_{x\to 1}\frac{(x+1)(x-1)}{x-1}$$
$$=2$$

Alternative Definition of Derivative -- Used for finding $f'(a)$: $f'(a)=\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$

a is a constant

6. $y=\frac{1}{x}$ at $x=7$

$$y'(7)=\lim_{x\to 7}\frac{\frac{1}{x}-\frac{1}{7}}{x-7}\cdot\frac{7x}{7x}$$
$$=\lim_{x\to 7}\frac{7-x}{(x-7)(7x)}$$
$$=\lim_{x\to 7}\frac{-1}{7x}$$
$$=-\frac{1}{49}$$

7. State the three reasons a function is not differentiable at $x=a$.

f is not differentiable at $x=a$ if

1) f is not continuous at $x=a$.
2) f has a cusp at $x=a$. (Slope from the left $\neq$ Slope from the right.)
3) f has a vertical slope at $x=a$.

For the following functions, state any values of x at which the function is not differentiable.

8. $f(x) = |5x - 2|$ f is not differentiable at $x = \frac{2}{5}$

(f has a cusp there.)

9. $f(x) = \dfrac{x^2 - x - 2}{x - 2}$ f is not differentiable at $x = 2$

(f is not continuous there)

10. $f(x) = \dfrac{1}{2x + 1}$ f is not differentiable at $x = -\frac{1}{2}$

(f is not continuous there)

11. $f(x) = x^{1/3}$

$f'(x) = \frac{1}{3}x^{-2/3}$

$f'(x) = \dfrac{1}{3x^{2/3}}$

This is undefined when $x = 0$

So the slope is vertical there → f is not differentiable at $x = 0$

(f has a vertical slope there)

Find the derivative of each using the Power Rule for Derivatives.

12. $f(x) = x^3 + 5x^2 - 3x + 8$

$f'(x) = 3x^2 + 10x - 3$

13. $f(x) = \dfrac{1}{x^5} = x^{-5}$

$f'(x) = \boxed{-5x^{-6}}$ Safe Stop answer

$= -\dfrac{5}{x^6}$ Multiple Choice answer

14. $f(x) = \sqrt{x} - 6\sqrt[3]{x} = x^{\frac{1}{2}} - 6x^{\frac{1}{3}}$

$f'(x) = \boxed{\frac{1}{2}x^{-\frac{1}{2}} - 2x^{-\frac{2}{3}}}$ SS

$= \dfrac{1}{2\sqrt{x}} - \dfrac{2}{x^{2/3}}$ mc

15. $y = x^3 + \cos x$

$y' = 3x^2 - \sin x$

16. $f(\theta) = \dfrac{\pi}{2}\sin\theta - \sec\theta$

$f'(\theta) = \frac{\pi}{2}\cos\theta - \sec\theta\tan\theta$

17. $y = \dfrac{1}{x} - 3\tan x = x^{-1} - 3\tan x$

$y' = \boxed{-x^{-2} - 3\sec^2 x}$ SS

$= -\dfrac{1}{x^2} - 3\sec^2 x$ mc

18. $f(t) = t^2 - \dfrac{4}{t^3} = t^2 - 4t^{-3}$

$f'(t) = \boxed{2t + 12t^{-4}}$

$= 2t + \dfrac{12}{t^4}$

19. $y = \dfrac{x^3 - 3x^2 + 4}{x^2} = x - 3 + 4x^{-2}$

$y' = \boxed{1 - 8x^{-3}}$

$= 1 - \dfrac{8}{x^3}$

Write the equation of the line tangent to each curve at the given point.

20. $y = x^4 - 3x^2 + 2$ at $(1,0)$

Point: $(1,0)$

Slope: $y' = 4x^3 - 6x$

$y'(1) = 4(1) - 6 = -2$

EQ of Tang. Line: $y - 0 = -2(x-1)$

21. $f(x) = \frac{2}{\sqrt[4]{x^3}}$ at $(1,2)$ $f(x) = 2x^{-\frac{3}{4}}$

Point: $(1,2)$

Slope: $f'(x) = -\frac{3}{2}x^{-\frac{7}{4}}$

$f'(1) = -\frac{3}{2}$

EQ of Tang. Line: $y - 2 = -\frac{3}{2}(x-1)$

Determine the points at which the graph of the function has a horizontal tangent line.

22. $y = x + \sin x$, $0 \le x \le 2\pi$

Horizontal Line $\Rightarrow$ Slope is 0

$y' = 1 + \cos x \overset{set}{=} 0$

$\cos x = -1$

$x = \pi$

$y = \pi + \sin\pi$

$y = \pi + 0$

$y = \pi$

Horizontal Tangent Line at the point (π, π)

State the function whose derivative is given by each limit.

23. $\lim_{\Delta x \to 0} \frac{\sqrt{x+\Delta x} - \sqrt{x}}{\Delta x} =$

$f(x) = \sqrt{x}$

24. $\lim_{h \to 0} \frac{\cos(x+h) + \cos x}{h} =$

$f(x) = \cos x$

25. $\lim_{h \to 0} \frac{\left[3(x+h)^2 - 1\right] - (3x^2 - 1)}{h} =$

$f(x) = 3x^2 - 1$

State the value of each limit.

26. $\lim_{h \to 0} \frac{\tan\left(\frac{\pi}{6} + h\right) - \frac{\sqrt{3}}{3}}{h} =$

$f(x) = \tan x$

$f'(x) = \sec^2 x$

$f'(\frac{\pi}{6}) = \boxed{\sec^2 \frac{\pi}{6}}$ SS

$= \frac{1}{\cos^2 \frac{\pi}{6}}$

$= \frac{1}{\left(\frac{\sqrt{3}}{2}\right)^2}$

$= \frac{1}{\frac{3}{4}} = \frac{4}{3}$ mc

27. $\lim_{\Delta x \to 0} \frac{(4+\Delta x)^{3/2} - 8}{\Delta x} =$

$f(x) = x^{3/2}$

$f'(x) = \frac{3}{2}x^{\frac{1}{2}}$

$f'(4) = \frac{3}{2}\sqrt{4}$

$= \frac{3}{2}(2)$

$= 3$

28. $\lim_{h \to 0} \frac{\cos(\pi + h) + 1}{h} =$

$f(x) = \cos x$

$f'(x) = -\sin x$

$f'(\pi) = -\sin\pi$

$= 0$