AP Calculus BC Name: Key Period:

Unit 1 – Wkst 2 – Continuity, IVT, Squeeze Thm

In #1-6, state any values of x at which the function is not continuous.

1. $f(x) = -3x^2 + 7$

f(x) is continuous for all x-values.

(All polynomials are continuous.)

2. $f(x) = \dfrac{4x^2 + 7x - 2}{x+2}$

f(x) is not continuous at $x = -2$.

(f(x) has a vertical asymptote at $x = -2$)

3. $f(x) = \tan x, -2\pi \le x \le 2\pi$

f(x) is not continuous at $x = -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}$

(f has vertical asymptotes at those x-values.)

4. $f(x) = \begin{cases} \dfrac{x^2 - 3x + 2}{x-2}, & x \ne 2 \\ 0, & x = 2 \end{cases}$

$\dfrac{(x-2)(x-1)}{x-2} \to 1$ as $x \to 2$

$f(2) = 0 \ne 1$ so f is not continuous at $x = 2$

5. $f(x) = \begin{cases} -2x + 3, & x < 1 \\ x^2, & x \ge 1 \end{cases}$

$-2(1) + 3 = 1$

$(1)^2 = 1$

$1 = 1$

f is continuous for all x-values

6. $f(x) = \begin{cases} \dfrac{1}{2}x + 1, & x \le 2 \\ 3 - x, & x > 2 \end{cases}$

$\frac{1}{2}(2) + 1 = 2$

$3 - 2 = 1$

$2 \ne 1$

f is not continuous at $x = 2$

7. Complete the sentence: "$f(x)$ is continuous at $x = a$ if …"

$\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = f(a)$

8. Is the discontinuity in #4 removable or non-removable?

Removable

9. Is the discontinuity in #6 removable or non-removable?

Non-Removable

10. Complete the sentence: "$f(x)$ has a removable discontinuity at $x = a$ if …"

$\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) \ne f(a)$

In #11-13 a) Verify that the IVT applies, and b) Find the value of c guaranteed by the IVT

11. $f(x) = x^3 - x^2 + x - 2$

on $[0,3]$, $f(c) = 4$

a) • $\left.\begin{array}{l} f(0) = -2 \\ f(3) = 19 \end{array}\right\}$ $f(0) < 4 < f(3)$

• f(x) is continuous $\forall x$

• By IVT $\exists$ c in $0 < c < 3 \ni f(c) = 4$

b) $x^3 - x^2 + x - 2 = 4$

check $x = 1$: $1 - 1 + 1 - 2 = 4$ NO

check $x = 2$: $8 - 4 + 2 - 2 = 4$ yes

$c = 2$

12. $f(x) = (x-3)^2 + 2$

on $[1,4]$, $f(c) = 5$

a) • $\left.\begin{array}{l} f(1) = 6 \\ f(4) = 3 \end{array}\right\}$ $f(4) < 5 < f(1)$

• f(x) is continuous for all x

• By IVT $\exists$ c in $1 < c < 4 \ni f(c) = 5$

b) $(x-3)^2 + 2 = 5$

$(x-3)^2 = 3$

$x - 3 = \pm\sqrt{3}$

$x = 3 \pm \sqrt{3}$

$3 + \sqrt{3}$ is not in the interval

$c = 3 - \sqrt{3}$

13. The height of a tree at time t is given by a continuous function H. Selected values of $H(t)$ are given in the table on the right. Explain why there must be a time at which the height of the tree is 12 meters.

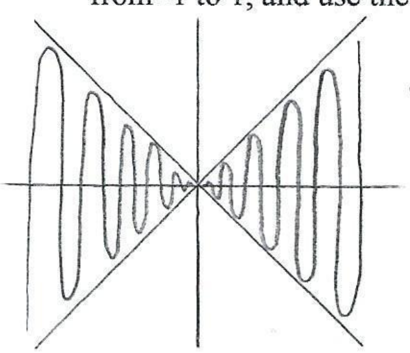
t (years)	2	3	5	7	10
$H(t)$ (meters)	1.5	2	6	11	15

- $\left.\begin{array}{l} H(7)=11 \\ H(10)=15 \end{array}\right\} H(7)<12<H(10)$
- H is given to be continuous
- By IVT, $\exists\, t,\ 7<t<10,\ \ni H(t)=12$

14. Without graphing, explain why you know that the function $f(x) = x^2 - 4x + 3$ has a zero in the interval [2,4].

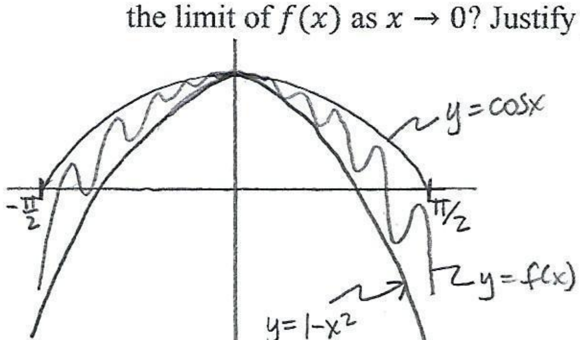
- $\left.\begin{array}{l} f(2)=-1 \\ f(4)=3 \end{array}\right\} f(2)<0<f(4)$
- $f(x)$ is continuous
- By IVT, $\exists\, x,\ 2<x<4,\ \ni f(x)=0$

15. On your calculator, graph $y = x$, $y = -x$, and $y = x\cos\left(\frac{50\pi}{x}\right)$ on the same graph over the x-interval from -1 to 1, and use the Squeeze Theorem to find $\lim\limits_{x\to 0}\left[x\cos\left(\frac{50\pi}{x}\right)\right]$.



- $y=x\cos\left(\frac{50\pi}{x}\right)$ is always between $y=x$ and $y=-x$.
- $\lim\limits_{x\to 0} x=0$ and $\lim\limits_{x\to 0} -x=0$
- By Squeeze Thm, $\lim\limits_{x\to 0} x\cos\left(\frac{50\pi}{x}\right)=0$

16. Sketch the graphs of $y = 1 - x^2$, $y = \cos x$, and $y = f(x)$, where f is any continuous function that satisfies the inequality $1 - x^2 \le f(x) \le \cos x$ for all x in the interval $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$. What can you say about the limit of $f(x)$ as $x \to 0$? Justify your reasoning.



- $1-x^2 \le f(x) \le \cos x$
- $\lim\limits_{x\to 0} 1-x^2=1$ and $\lim\limits_{x\to 0} \cos x=1$
- By Squeeze Thm, $\lim\limits_{x\to 0} f(x)=1$

17. If $1 \le f(x) \le x^2 + 2x + 2$ for all x, find $\lim\limits_{x\to -1} f(x)$.

$\lim\limits_{x\to -1} 1=1$ and $\lim\limits_{x\to -1}(x^2+2x+2)=1$

By Squeeze Thm $\lim\limits_{x\to -1} f(x)=1$