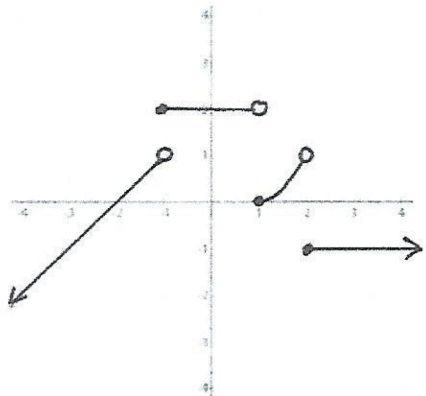
AP Calculus BC
Unit 1 – Wkst 1 – Limits Review

Name: Key Period_____

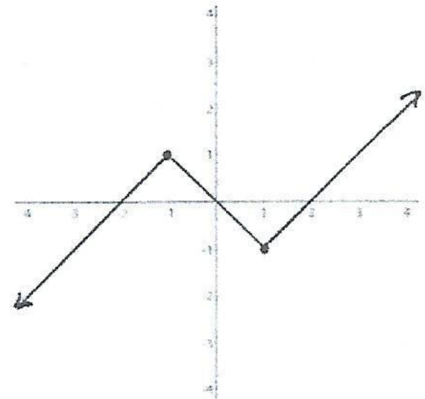
Show all work. No calculator.

Given the graphs of functions f and g, evaluate each, if it exists. Write DNE if the limit does not exist.

$f(x)$



$g(x)$



1. $\lim_{x\to 2^-} f(x) = \boxed{1}$

2. $\lim_{x\to 2^+} f(x) = \boxed{-1}$

3. $\lim_{x\to 2} f(x) = \boxed{DNE}$

   $\lim_{x\to 2^-} f(x) \neq \lim_{x\to 2^+} f(x)$

4. $f(2) = \boxed{-1}$

5. $\lim_{x\to -1^-} f(x) = \boxed{1}$

6. $\lim_{x\to -1^+} f(x) = \boxed{2}$

7. $\lim_{x\to -1} f(x) = \boxed{DNE}$

   $\lim_{x\to -1^-} f(x) \neq \lim_{x\to -1^+} f(x)$

8. $f(-1) = \boxed{2}$

9. $\lim_{x\to 1^-} g(x) = \boxed{-1}$

10. $\lim_{x\to 1^+} g(x) = \boxed{-1}$

11. $\lim_{x\to 1} g(x) = \boxed{-1}$

    $\lim_{x\to 1^-} g(x) = \lim_{x\to 1^+} g(x) = -1$

12. $g(1) = \boxed{-1}$

13. $\lim_{x\to \infty} f(x) = \boxed{-1}$

    as $x\to\infty$, $y\to -1$

14. $\lim_{x\to -\infty} f(x) = -\infty$ $\boxed{DNE}$

    as $x\to -\infty$, $y\to -\infty$

15. $\lim_{x\to \infty} g(x) = \infty$ $\boxed{DNE}$

    as $x\to\infty$, $y\to\infty$

16. $\lim_{x\to -\infty} g(x) = -\infty$ $\boxed{DNE}$

    as $x\to -\infty$, $y\to -\infty$

17. $\lim_{x\to -1}[f(x)+g(x)] = \boxed{DNE}$

    $\downarrow$ DNE $\quad \downarrow$ 1

    If either is DNE, the entire limit is DNE

18. $\lim_{x\to 0}[2f(x)+3g(x)] =$

    $= 2\lim_{x\to 0} f(x) + 3\lim_{x\to 0} g(x)$

    $= 2(2) + 3(0)$

    $= \boxed{4}$

19. $\lim_{x\to -1}[f(x)g(x)] = \boxed{DNE}$

    $\downarrow$ DNE $\quad \downarrow$ 1

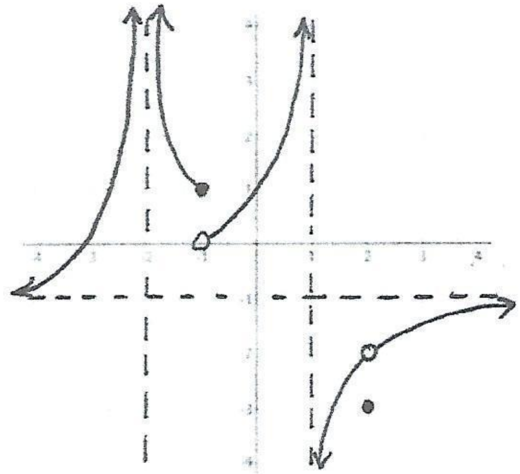
    If either is DNE, the entire limit is DNE

20. $\lim_{x\to 0} \dfrac{f(x)}{g(x)} = \boxed{DNE}$

    $= \dfrac{\lim_{x\to 0} f(x)}{\lim_{x\to 0} g(x)}$

    $= \dfrac{2}{0}$ DNE

Use the figure at the right to evaluate each limit, if it exists.



Graph of $f(x)$

21. $\lim_{x\to 1^+} f(x) =$ $-\infty$ $\boxed{DNE}$

22. $\lim_{x\to 1^-} f(x) =$ ∞ $\boxed{DNE}$

23. $\lim_{x\to 1} f(x) = \boxed{DNE}$

If either one-sided limit is DNE, the limit is DNE

24. $\lim_{x\to 2^-} f(x) = \boxed{-2}$

25. $\lim_{x\to 2^+} f(x) = \boxed{-2}$

26. $\lim_{x\to 2} f(x) = \boxed{-2}$

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) = -2$

27. $f(2) = \boxed{-3}$

28. $\lim_{x\to -1^+} f(x) = \boxed{0}$

29. $\lim_{x\to \infty} f(x) = \boxed{-1}$

as $x\to\infty$, $y\to -1$

30. $\lim_{x\to -\infty} f(x) = \boxed{-1}$

as $x\to\infty$, $y\to -1$

Find each limit, if it exists. If the limit does not exist write DNE.

31. $\lim_{t\to -4} \frac{t^2-4}{t-4} = \boxed{\frac{16-4}{-8}}$ SS

$= \frac{12}{-8}$

$= -\frac{3}{2}$ mc answer

32. $\lim_{x\to \frac{5\pi}{3}} \frac{\tan x}{\cos x} = \boxed{\frac{-\sqrt{3}}{\frac{1}{2}}}$ SS

$= -2\sqrt{3}$ mc answer

33. $\lim_{x\to -3} \frac{x^4-81}{3x+9} =$ ($\frac{0}{0}$ Indeterminate)

$= \lim_{x\to -3} \frac{(x^2+9)(x^2-4)}{3(x+3)}$

$= \lim_{x\to -3} \frac{(x^2+9)(x-3)\cancel{(x+3)}}{3\cancel{(x+3)}}$

$= \boxed{\frac{18(-6)}{3}} = -36$ mc

34. $\lim_{x\to 3} \frac{x-3}{x^3-27} =$ $\left(\frac{0}{0}\right)$

$= \lim_{x\to 3} \frac{\cancel{x-3}}{\cancel{(x-3)}(x^2+3x+9)}$

$= \boxed{\frac{1}{9+9+9}} = \frac{1}{27}$

35. $\lim_{x\to 0} \frac{\sqrt{x+2}-\sqrt{2}}{x} =$ $\left(\frac{0}{0}\right)$

$= \lim_{x\to 0} \left(\frac{\sqrt{x+2}-\sqrt{2}}{x}\right)\left(\frac{\sqrt{x+2}+\sqrt{2}}{\sqrt{x+2}+\sqrt{2}}\right)$

mult by conjugate

$= \lim_{x\to 0} \frac{x+2-2}{x(\sqrt{x+2}+\sqrt{2})}$

$= \lim_{x\to 0} \frac{\cancel{x}}{\cancel{x}(\sqrt{x+2}+\sqrt{2})} = \boxed{\frac{1}{2\sqrt{2}}}$

36. $\lim_{x\to 0} \left[\frac{\frac{1}{x+1}-1}{x}\right] \cdot \left(\frac{x+1}{x+1}\right)$

mult by complex denom.

$= \lim_{x\to 0} \frac{1-(x+1)}{x(x+1)}$

$= \lim_{x\to 0} \frac{-x}{x(x+1)}$

$= \lim_{x\to 0} \frac{-1}{x+1} = \boxed{-1}$

37. $\lim_{x\to 0} \left(\frac{1-\cos x}{\sin x}\right) \cdot \left(\frac{1+\cos x}{1+\cos x}\right)$

$= \lim_{x\to 0} \frac{1-\cos^2 x}{\sin x(1+\cos x)}$ $\leftarrow$ $\cos^2 x + \sin^2 x = 1$, $1-\cos^2 x = \sin^2 x$

$= \lim_{x\to 0} \frac{\sin^2 x}{\sin x(1+\cos x)}$

$= \lim_{x\to 0} \frac{\sin x}{1+\cos x} = \frac{0}{2} = \boxed{0}$

38. $\lim_{x\to \pi} \left(\frac{1+\cos x}{\sin x}\right) \cdot \left(\frac{1-\cos x}{1-\cos x}\right)$

$= \lim_{x\to \pi} \frac{1-\cos^2 x}{\sin x(1-\cos x)}$

$= \lim_{x\to \pi} \frac{\sin^2 x}{\sin x(1-\cos x)}$

$= \lim_{x\to \pi} \frac{\sin x}{1-\cos x} = \frac{0}{2} = \boxed{0}$

39. $\lim_{x\to \frac{\pi}{2}} \frac{1-\cos x}{\sin x} = \frac{1}{1} = \boxed{1}$

40. $\lim_{x\to2}\left(\frac{\sqrt{11-x}-3}{x-2}\right)\left(\frac{\sqrt{11-x}+3}{\sqrt{11-x}+3}\right)$

$=\lim_{x\to2}\frac{11-x-9}{(x-2)(\sqrt{11-x}+3)}$

$=\lim_{x\to2}\frac{2-x}{(x-2)(\sqrt{11-x}+3)}$

$=\lim_{x\to2}\frac{-1}{\sqrt{11-x}+3}=\boxed{\frac{-1}{6}}$

41. $\lim_{x\to0}\left(\frac{\frac{2}{x+1}-\frac{4}{2-x}}{x}\right)\left(\frac{(x+1)(2-x)}{(x+1)(2-x)}\right)$

$=\lim_{x\to0}\frac{2(2-x)-4(x+1)}{x(x+1)(2-x)}$

$=\lim_{x\to0}\frac{4-2x-4x-4}{x(x+1)(2-x)}$

$=\lim_{x\to0}\frac{-6}{(x+1)(2-x)}=\frac{-6}{2}=\boxed{-3}$

42. $\lim_{x\to-3}\left(\frac{\frac{1}{x+4}-1}{x+3}\right)\left(\frac{x+4}{x+4}\right)$

$=\lim_{x\to-3}\frac{1-(x+4)}{(x+3)(x+4)}$

$=\lim_{x\to-3}\frac{-x-3}{(x+3)(x+4)}$

$=\lim_{x\to-3}\frac{-1}{x+4}=\frac{-1}{1}=\boxed{-1}$

43. $\lim_{x\to\infty}\frac{9x+2-4x^3}{5+x^2+7x^4}=$

$=\lim_{x\to\infty}\frac{-4x^3}{7x^4}$

$=\lim_{x\to\infty}\frac{-4}{7x}=\boxed{0}$

44. $\lim_{x\to\infty}\frac{3x^2-4x^3}{5x^3+2x}=$

$=\lim_{x\to\infty}\frac{-4x^3}{5x^3}$

$=\lim_{x\to\infty}\frac{-4}{5}=\boxed{-\frac{4}{5}}$

45. $\lim_{x\to\infty}\frac{3x^5-4x^3}{7x^3-9x}=$

$=\lim_{x\to\infty}\frac{3x^5}{7x^3}$

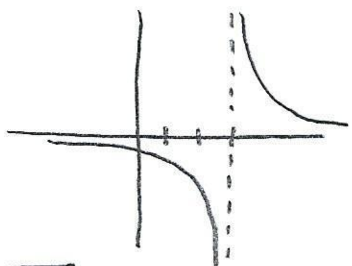
$=\lim_{x\to\infty}\frac{3x^2}{7}=\frac{\infty}{7}=\infty$ $\boxed{DNE}$

46. $\lim_{x\to3^-}\frac{1}{x-3}=-\infty$ $\boxed{DNE}$

47. $\lim_{x\to3^+}\frac{1}{x-3}=\infty$ $\boxed{DNE}$

48. $\lim_{x\to3}\frac{1}{x-3}=\boxed{DNE}$

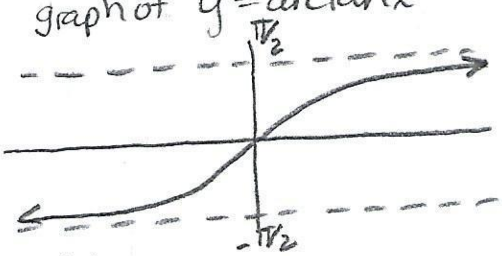
Graph of $y=\frac{1}{x-3}$:



49. $\lim_{x\to\infty}\arctan x=\boxed{\frac{\pi}{2}}$

50. $\lim_{x\to-\infty}\arctan x=\boxed{-\frac{\pi}{2}}$

graph of $y=\arctan x$



51. $\lim_{x\to\infty}\frac{\sin x}{x}=\boxed{0}$

Think:
as $x\to\infty$, $\sin x$ oscillates from -1 to 1
But the denominator is going to ∞.
Any number between -1 and 1 that is divided by a huge number, goes to 0.

52. $\lim_{x\to\infty}\cos\left(\frac{1}{x}\right)=\cos 0=\boxed{1}$

as $x\to\infty$, $\frac{1}{x}\to0$

53. $\lim_{x\to0}\cos\left(\frac{1}{x}\right)=\boxed{DNE}$

as $x\to0$, $\frac{1}{x}\to\infty$
as $\frac{1}{x}$ goes to ∞,
$\cos\frac{1}{x}$ oscillates from
-1 to 1.
So this limit
does not exist.

54. $\lim_{x\to0^+}\frac{10\cos x}{\sin x}=\frac{1}{0^+}=\infty$ $\boxed{DNE}$