

Unit 1 – Wkst 2 – Continuity, IVT, Squeeze Thm

In #1-6, state any values of x at which the function is not continuous.

1. $f(x) = -3x^2 + 7$

2. $f(x) = \frac{4x^2 + 7x - 2}{x + 2}$

3. $f(x) = \tan x, -2\pi \leq x \leq 2\pi$

4. $f(x) = \begin{cases} \frac{x^2 - 3x + 2}{x - 2}, & x \neq 2 \\ 0, & x = 2 \end{cases}$

5. $f(x) = \begin{cases} -2x + 3, & x < 1 \\ x^2, & x \geq 1 \end{cases}$

6. $f(x) = \begin{cases} \frac{1}{2}x + 1, & x \leq 2 \\ 3 - x, & x > 2 \end{cases}$

7. Complete the sentence: “ $f(x)$ is continuous at $x = a$ if ...”

8. Is the discontinuity in #4 removable or non-removable?

9. Is the discontinuity in #6 removable or non-removable?

10. Complete the sentence: “ $f(x)$ has a removable discontinuity at $x = a$ if ...”

In #11-13 a) Verify that the IVT applies, and b) Find the value of c guaranteed by the IVT

11. $f(x) = x^3 - x^2 + x - 2$

on $[0,3]$, $f(c) = 4$

12. $f(x) = (x - 3)^2 + 2$

on $[1,4]$, $f(c) = 5$

13. The height of a tree at time t is given by a continuous function H . Selected values of $H(t)$ are given in the table on the right. Explain why there must be a time at which the height of the tree is 12 meters.

t (years)	2	3	5	7	10
$H(t)$ (meters)	1.5	2	6	11	15

14. Without graphing, explain why you know that the function $f(x) = x^2 - 4x + 3$ has a zero in the interval $[2,4]$.

15. On your calculator, graph $y = x$, $y = -x$, and $y = x \cos\left(\frac{50\pi}{x}\right)$ on the same graph over the x -interval from -1 to 1, and use the Squeeze Theorem to find $\lim_{x \rightarrow 0} \left[x \cos\left(\frac{50\pi}{x}\right) \right]$.

16. Sketch the graphs of $y = 1 - x^2$, $y = \cos x$, and $y = f(x)$, where f is any continuous function that satisfies the inequality $1 - x^2 \leq f(x) \leq \cos x$ for all x in the interval $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$. What can you say about the limit of $f(x)$ as $x \rightarrow 0$? Justify your reasoning.

17. If $1 \leq f(x) \leq x^2 + 2x + 2$ for all x , find $\lim_{x \rightarrow -1} f(x)$.