Summer Packet for Students Entering AP Calculus BC

This packet is a review of some of the concepts you are expected to know upon entering AP Calculus BC. The course begins where Pre-AP Precalculus BC stopped in May. You will NOT be turning this packet in for a grade, however, your teacher expects that you know this material. It will not be re-taught. No calculator.

I. Trigonometry

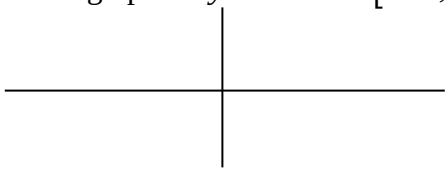
You must know all of the unit circle values for all six trig functions (no calculator).

1. $\sin\frac{\pi}{6}=$ 2. $\cos\frac{4\pi}{3}=$ 3. $\tan\left(\frac{5\pi}{4}\right)=$ 4. $\sec\frac{\pi}{6}=$ 5. $\csc\frac{5\pi}{3}=$

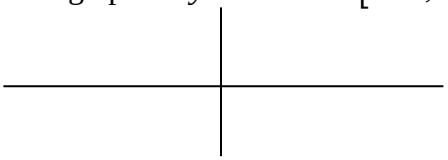
6. $\sin\pi=$ 7. $\tan 0=$ 8. $\cos\frac{3\pi}{2}=$ 9. $\sec\frac{\pi}{3}=$ 10. $\cot\frac{\pi}{2}=$

You must know the basic graphs of all six trig functions.

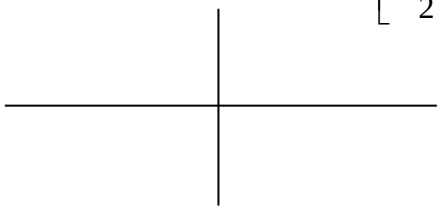
11. Sketch the graph of $y=\sin x$ on $[-2\pi,2\pi]$



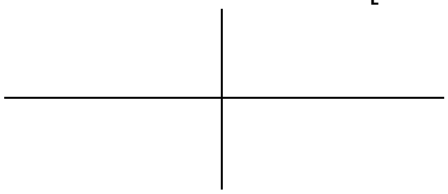
12. Sketch the graph of $y=\cos x$ on $[-2\pi,2\pi]$



13. Sketch the graph of $y=\tan x$ on $\left[-\frac{\pi}{2},\frac{3\pi}{2}\right]$



14. Sketch the graph of $y=\cot x$ on $[-\pi,\pi]$



You must know the basic trig identities:

Pythagorean Identities

$\sin^2 x+\cos^2 x=1$

$\tan^2 x+1=\sec^2 x$

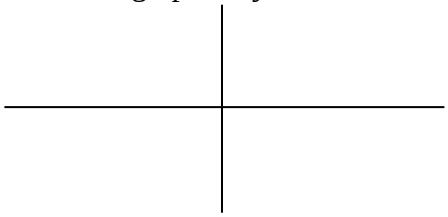
$\cot^2 x+1=\csc^2 x$

Double Angle Identities

$\sin 2x=2\sin x\cos x$

$\cos 2x=\cos^2 x-\sin^2 x$

Know the graph of $y=\arctan x$:



You must be able to solve trig equations. Solve each equation. Give solutions from $[0,2\pi]$.

16. $2\sin x+3=4$ 17. $2\cos^2 x-5\cos x+2=0$ 18. $2\sin 2x+1=0$

Know how to obtain values of the inverse trig functions

19. $\arcsin\left(\frac{1}{2}\right)=$ 20. $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)=$ 21. $\arctan(1)=$ 22. $\cot^{-1}(0)=$

23. $\arccos\frac{\sqrt{2}}{2}=$ 24. $\cot^{-1}\left(-\sqrt{3}\right)=$ 25. $\csc^{-1}\left(-\sqrt{2}\right)=$ 26. $\arctan\left(-\frac{\sqrt{3}}{3}\right)=$

II. Functions

Know the graphs of all of the parent functions you've learned. Sketch each graph. (No calculator.)

27. $y = x$ 28. $y = x^2$ 29. $y = x^3$ 30. $y = \sqrt{x}$ 31. $y = \frac{1}{x}$

32. $y = |x|$ 33. $y = [\![x]\!]$ 34. $y = e^x$ 35. $y = \ln x$ 36. $y = \sqrt[3]{x}$

Know how to write the equation of a line given the slope and a point.

37. Write an equation of the line with slope $\frac{3}{4}$ going through the point $(-2, 7)$.

Dividing polynomials.

38. Divide $\frac{x^3 + 2x^2 - 5x - 6}{x^2 + 3x - 1} =$

39. Divide using synthetic division $\frac{x^4 + 5x^3 - 2x - 8}{x + 3} =$

Be able to solve various equations and inequalities.

40. $x^2 - 5x + 6 = 0$ 41. $\frac{3}{x+2} - \frac{6x}{x^2 - 4} = 0$ 42. $(3x - 1)^{3/2} = 64$

43. $\left|\frac{x-4}{3}\right| < 1$ 44. $e^{2x-1} = 4$ 45. $\ln\left(\frac{4x+1}{3}\right) = 2$

Be able to expand logarithms.

46. $\ln\frac{(x+1)^2}{\sqrt{x-3}} =$

Be able to condense logarithms.

47. $3\ln 2 + \ln x - \frac{1}{2}\ln(x+1) =$

IV. Polar Coordinates

Remember: $x = r\cos\theta$ $y = r\sin\theta$ $r^2 = x^2 + y^2$ $\tan\theta = \frac{y}{x}$

48. Write the rectangular point $(-3, 3)$ in polar form.

49. Write the polar point $\left(4, \pi/3\right)$ in rectangular form.

Sketch each graph (Remember - No Calculator).

50. $r = 3\sin\theta$ 51. $r = 4\cos 3\theta$ 52. $r = 2\cos 4\theta$ 53. $r = 3 + 3\sin\theta$

54. $r = 2 + 3\cos\theta$ 55. $r = 3 + 2\sin\theta$ 56. $r = 4 - 3\cos\theta$ 57. $r = -2 - 3\sin\theta$

III. Calculus Concepts

Limits: Be able to evaluate basic limits.

58. $\lim\limits_{x\to 2}\dfrac{x^2-3x+1}{x+2} =$ 59. $\lim\limits_{x\to 2}\dfrac{x^2+6x-16}{x-2} =$ 60. $\lim\limits_{x\to 0}\dfrac{\sqrt{16+x}-4}{x} =$

61. $\lim\limits_{x\to\infty}\dfrac{5x^3-3x+1}{7x^3+2} =$ 62. $\lim\limits_{x\to 2^+}\dfrac{1}{x-2} =$ 63. $\lim\limits_{x\to\infty}\dfrac{\sin x}{x} =$

Continuity:

Know the definition of continuity and how to apply it.

A function $f(x)$ is continuous at $x = a$ if and only if $\lim\limits_{x\to a^-} f(x) = \lim\limits_{x\to a^+} f(x) = f(a)$

64. Find the value(s) of k so that f will be continuous at $x = 2$ given $f(x) = \begin{cases} kx^2 & , x \le 2 \\ 2x + k & , x > 2 \end{cases}$

Derivatives:

Know how to find the derivative of a function using the definition of derivative.

65. Given $f(x) = x^2 - 3$, which of the following is $f'(x)$, the derivative of $f(x)$?

(A) $\lim\limits_{h\to 0}\dfrac{\left((x+h)^2-3\right)-\left(x^2-3\right)}{h}$ (B) $\lim\limits_{x\to 0}\dfrac{\left((x+h)^2-3\right)-\left(x^2-3\right)}{h}$

(C) $\lim\limits_{h\to 0}\dfrac{\left(x^2-3\right)-\left((x+h)^2-3\right)}{h}$ (D) $\lim\limits_{x\to 0}\dfrac{\left(x^2-3\right)-(3)}{x-3}$

Know how to find the derivative using the power rule, product rule, quotient rule, and chain rule.

Find the derivative of each. Use proper notation.

66. $f(x) = 3x^4 + 2x^3 - 4x^2 + 7$ 67. $y = \sqrt[4]{x^3} + 7x$ 68. $h(x) = \sec x$

69. $f(x) = x^2\sin x$ 70. $g(x) = \dfrac{x}{\cos x}$ 71. $f(x) = \tan(3x)$

72. $y = 4\sin\left(3x^3\right)$ 73. $g(x) = 5\cos^3(4x)$ 74. $y = \sqrt[3]{4x^3 + 3x}$