Key

Dear AP-Calculus students,

AP Calculus is a capstone course for students who enjoy challenging mathematics. The course beautifully connects concepts from your previous coursework in Algebra, Geometry, Trigonometry, and Pre-Calclulus. Therefore, it is imperative that you come to class armed will all prior knowledge. This packet is designed to sharpen your skills and to prepare you for the calculus experience that is to come. All of the problems involve concepts and skills you should have already mastered and will encounter during the upcoming year.

Included you will find the worked-out solutions so that you can easily check your answers or work backwards to relearn content you have forgotten. The packet is a tool to prepare you for college-level coursework. The packet is NOT for a grade. Do the work you need to do in order to be prepared. For some of you, that means completing most of the packet. For others, you may just need to review a couple of concepts. Most of the problems do not require a calculator.

AP Calculus can best be summed up by two core ideas: hard work and integrity

Hard Work.
As your teacher, we will do our best to make difficult calculus ideas easy to understand, but at the end of the day, you need to do the work. If time is a legitimate concern for you, because you are now the captain of the team, squad leader, or the senior "in charge" of some important event, you need to be honest with yourself about your ability to get everything done well. For some of you, the AP Calculus AB course might be something to consider if you think you will be overwhelmed. For those of you who will be able to dedicate more time to your studies, the AP Calculus BC course might be the better fit.

Integrity.
Integrity is the essence of everything successful. R. Buckminster Fuller
Being enrolled in an advanced math class, the courses you take will be challenging. You may be tempted to take short cuts, even cheat on assignments. This may satisfy short-term goals, but will cause you strife in the long term. Be cognizant of your studies, put forth your best effort, and rise to the challenges ahead of you.

Here are some websites that might also help you get ready for calculus:
http://www.purplemath.com/modules/index.htm contains algebra and advanced algebra topics.

http://www.math.buffalo.edu/rur/rurci3.cgi has great review quizzes on pre-calculus topics.

https://www.khanacademy.org/math/precalculus has great video tutorials and problems.

See you in August,

The AP Calculus Team

Formula Sheet

Reciprocal Identities: $\csc x = \frac{1}{\sin x}$ $\sec x = \frac{1}{\cos x}$ $\cot x = \frac{1}{\tan x}$

Quotient Identities: $\tan x = \frac{\sin x}{\cos x}$ $\cot x = \frac{\cos x}{\sin x}$

Pythagorean Identities: $\sin^2 x + \cos^2 x = 1$ $\tan^2 x + 1 = \sec^2 x$ $1 + \cot^2 x = \csc^2 x$

Double Angle Identities: $\sin 2x = 2\sin x \cos x$

$$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$

$$\cos 2x = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x = 2\cos^2 x - 1$$

Logarithms:
$y = \log_a x$ is equivalent to $x = a^y$

Product property: $\log_b mn = \log_b m + \log_b n$

Quotient property: $\log_b \frac{m}{n} = \log_b m - \log_b n$

Power property: $\log_b m^p = p \log_b m$

Property of equality: If $\log_b m = \log_b n$,
then m = n

Change of base formula: $\log_a n = \frac{\log_b n}{\log_b a}$

Fractional exponent: $\sqrt[b]{x^e} = x^{\frac{e}{b}}$

Negative Exponents: $x^{-n} = 1/x^n$

The Zero Exponent: $x^0=1$, for x not equal to 0.

Multiplying Powers

Multiplying Two Powers of the Same Base:
$(x^a)(x^b) = x^{(a+b)}$

Multiplying Powers of Different Bases:
$(xy)^a = (x^a)(y^a)$

Dividing Powers

Dividing Two Powers of the Same Base:
$(x^a)/(x^b) = x^{(a-b)}$

Dividing Powers of Different Bases:
$(x/y)^a = (x^a)/(y^a)$

Slope-intercept form: $y = mx + b$

Point-slope form: $y = m(x - x_1) + y_1$

Standard form: $Ax + By + C = 0$

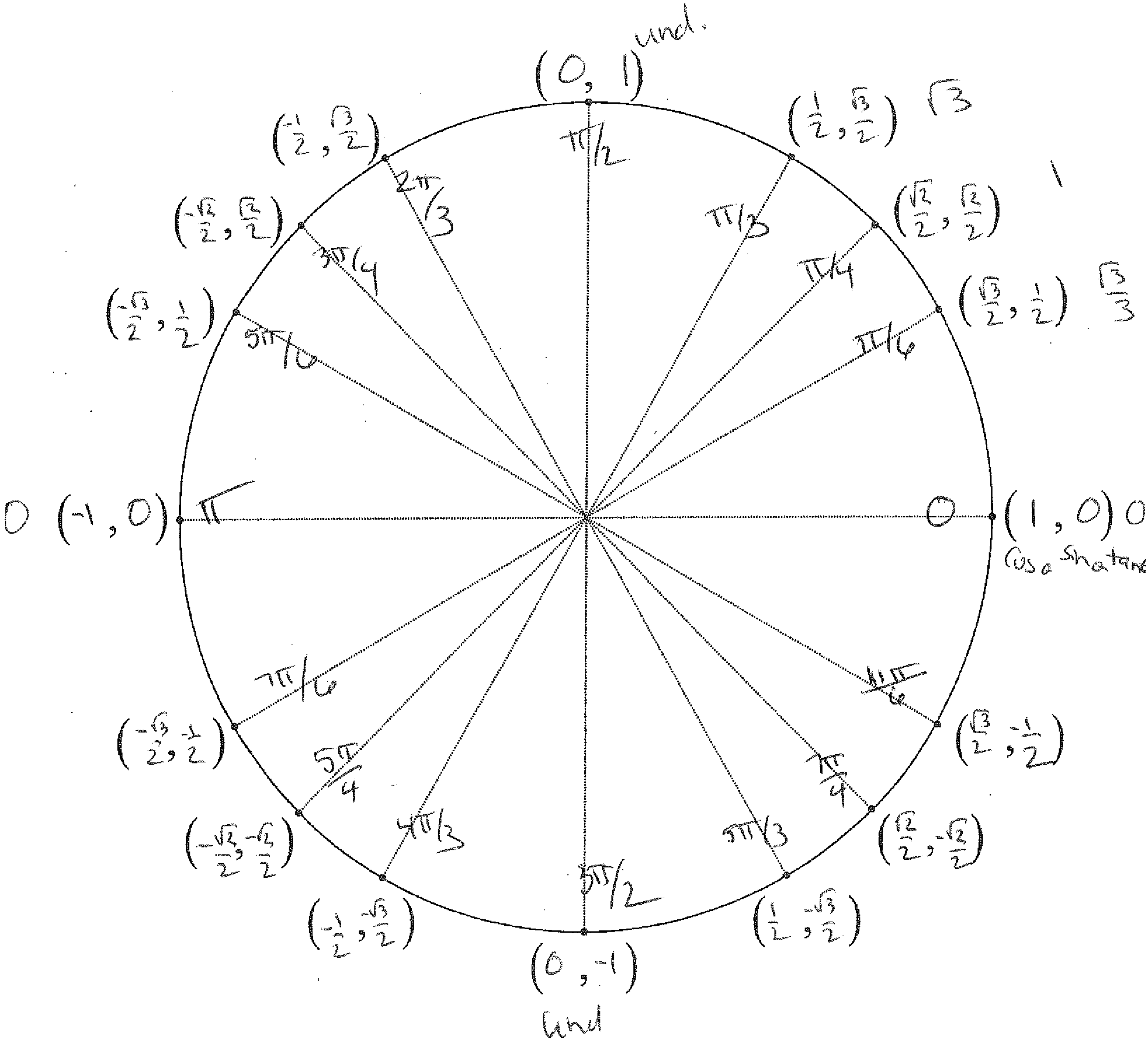
The circle is known as the **unit circle**, since its radius is one unit. It is an integral part of trigonometry.

You will cut out the triangles on the next page to help fill in the coordinates of this unit circle. Use what you know about reflections to help!

HINTS FOR QUADRANT II:
The angles in quadrant II are 120°, 135°, and 150°.

Look at the 120° point and the 60° point.
What should be true about their *y*-coordinates?

What should be true about their *x*-coordinates?



Trigonometry

Give the Exact Value of each without using a calculator.

1) $\tan\left(\frac{5\pi}{6}\right)$ $\frac{-\sqrt{3}}{3}$ 2) $\sin\left(\frac{5\pi}{3}\right)$ $\frac{-\sqrt{3}}{2}$ 3) $\sin\frac{5\pi}{6}+\cos\frac{5\pi}{3}$ $\frac{1}{2}+\frac{1}{2}=1$ 4) $\cot\left(\frac{11\pi}{6}\right)$ $-\sqrt{3}$ 5) $\sec\left(\frac{2\pi}{3}\right)$ -2

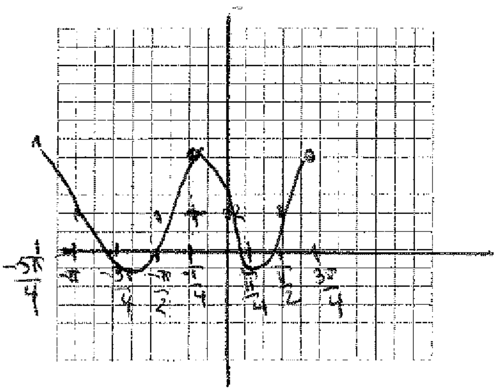
6) Graph $y=3\cos 2\left(x+\frac{\pi}{4}\right)+2$

amplitude = 3

period = $\frac{2\pi}{2}=\pi$

phase shift = left $\pi/4$

vertical shift = up 2



7) arctan 1 $\frac{\pi}{4}$ 8) $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ $\frac{5\pi}{6}$ 9) $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ $-\pi/3$ 10) $\arctan\left(-\sqrt{3}\right)$ $\frac{-\pi}{3}$

Trigonometric Identities

Given: $\sec\alpha=-\frac{17}{8}, \frac{\pi}{2}<\alpha<\pi$. 1. Find $\sin 2x$ 2. Find $\cos 2x$.

$2\sin\alpha\cos\alpha$
$2\left(\frac{15}{17}\right)\left(-\frac{8}{17}\right)=\frac{-240}{289}$

$\cos^2 x-\sin^2 x$
$\left(-\frac{8}{17}\right)^2-\left(\frac{15}{17}\right)^2$
$\boxed{\frac{-161}{289}}$

3) Prove: $\frac{1}{\cos x+1}+\frac{1}{\cos x-1}=-2\csc x\cot x$ 5) Simplify $\csc x\left(1-\cos^2 x\right)$

$\frac{\cos x-1+\cos x+1}{\cos^2 x-1}$
$\frac{2\cos x}{-\sin^2 x}$
$x\left(\frac{\cos x}{\sin x}\right)$
$\csc x\cot x$ ✓

$\csc x\left(\sin^2 x\right)$
$\sin x$

6) Use half angle formulas to find the exact value of $\tan\frac{7\pi}{8}$ $\tan\left(\frac{1}{2}\cdot\frac{7\pi}{4}\right)=\frac{1-\cos\frac{7\pi}{4}}{\sin\frac{7\pi}{4}}=\frac{1-\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}}$

$\frac{2-\sqrt{2}}{2}\cdot\left(\frac{-2}{\sqrt{2}}\right)=\frac{-2+\sqrt{2}}{\sqrt{2}}$ $1-\sqrt{2}$

Solve each trigonometric equations for all solutions from [0, 2π).

7) $\cos^3 x=\cos x$ 8) $2\sin^2 2x=1$ 9) $2\sin^2 x-7\sin x+3=0$

$\cos x\left(\cos^2 x-1\right)=0$
$\cos x\ (\cos x-1)(\cos x+1)=0$
$\left\{\frac{\pi}{2},\frac{3\pi}{2}, 0, \pi\right\}$

$\sin^2(2x)=\frac{1}{2}$
$\sin(2x)=\pm\sqrt{2}/2$
$\left\{\frac{\pi}{8},\frac{3\pi}{8},\frac{5\pi}{8},\frac{7\pi}{8},\frac{9\pi}{8},\frac{11\pi}{8},\frac{13\pi}{8},\frac{15\pi}{8}\right\}$

$(2\sin x-1)(\sin x-3)=0$
$\sin x=1/2$ $\sin x=3$
$x=\left\{\frac{\pi}{6},\frac{5\pi}{6}\right\}$

Functions

1) Write the equation of the line passing through (5, -6) and (12, -1)

$m=\frac{5}{7}$
$y+1=\frac{5}{7}(x-12)$
or
$y+6=\frac{5}{7}(x-5)$

2) Determine the domain $f(x)=\frac{1}{\sqrt{x^2-4}}$

$x^2-4>0$
+ − +
−2 2

$(-\infty,-2)\cup(2,\infty)$

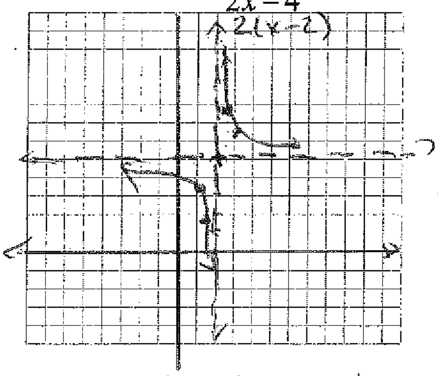
$x = \frac{4}{2y-5}$

$2y-5 = \frac{4}{x}$

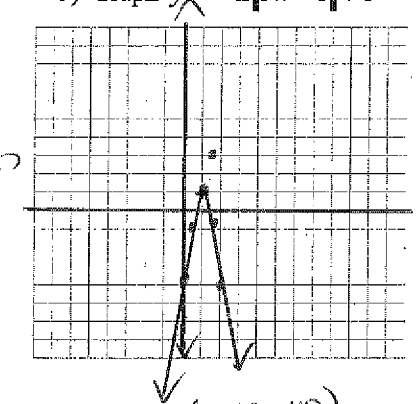
$2y = \frac{4}{x}+5$

3) Find the inverse: $f(x) = \dfrac{4}{2x-5}$ $\boxed{f^{-1}(x) = \frac{2}{x}+\frac{5}{2}}$

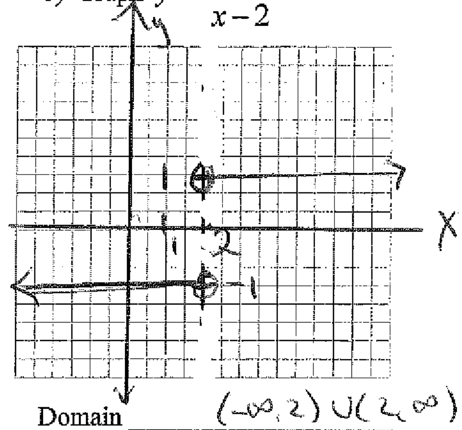
4) Graph $y = \dfrac{3}{2x-4}+5$ $2(x-2)$



5) Graph $y = -2\|3x-3\|+1$ $3(x-1)$



6) Graph $y = \dfrac{\|x-2\|}{x-2}$



Domain $(-\infty, 2)\cup(2,\infty)$ Domain $(-\infty, \infty)$ Domain $(-\infty, 2)\cup(2,\infty)$

Range $(-\infty, 5)\cup(5,\infty)$ Range $(-\infty, 1]$ Range $y = -1 \cup 1$

Polynomials

1) Solve $\dfrac{1}{x-2} = \dfrac{3}{x+2} - \dfrac{6x}{x^2-4}$

Factor each

$x+2 = 3x-6-6x$

$4x = -8$

$x = -2$ ext.

2) Solve $(3x-1)^{\frac{2}{3}} = 64$ $\frac{3}{2}$ $3/2$

$3x-1 = \pm 512$

$x \left\{171, -\frac{511}{3}\right\}$

3) $5(x+1)^2 = 60$

$x+1 = \pm\sqrt{12}$

$\boxed{x = -1 \pm 2\sqrt{3}}$

4) $4x^2(8y-3)^4 - 2x^3(8y-3)^5$

$2x^2(8y-3)^4(2-8xy+6x)$

5) $(x-5)^2 + 4(x-5) + 4$

$(x-3)^2$

6) $27y^3 + 64$ $(3y+4)(9y^2-12y+16)$

5) Divide using long division $\dfrac{x^3+2x^2-5x-6}{x^2+3x-1}$

$x-1$

$x^2+3x-1 \overline{)\, x^3+2x^2-5x-6}$

$-x^3-3x^2+x$

$-x^2-4x-6$

x^2+3x-1

$\boxed{x-1+\frac{-x-7}{x^2+3x-1}}$

6) Divide using synthetic division $\dfrac{x^4+5x^3-2x-8}{x+3}$

$-3 \mid 1 \quad 5 \quad 0 \quad -2 \quad -8$

$\quad -3 \quad -6 \quad 18 \quad -48$

$1 \quad 2 \quad -6 \quad 16 \quad -56$

$x^3+2x^2-6x+16+\frac{-56}{x+3}$

Solve each inequality

7) $\dfrac{x-2}{x-1} < 1$

$\frac{x-2-x+1}{x-1} < 0$

$\frac{-1}{x-1} < 0$

$+ \quad 0 \quad -$

1

$\boxed{(1,\infty)}$

8) $x^3 - x^2 - 3x < 0$ $(-\infty, \frac{1-\sqrt{13}}{2}) \cup (0, \frac{1+\sqrt{13}}{2})$

$x(x^2-x-3) < 0$

$\frac{1\pm\sqrt{1-4(-3)}}{2}$

$- \quad 0 \quad + \quad 0 \quad - \quad 0 \quad +$

$\frac{1-\sqrt{13}}{2} \quad 0 \quad \frac{1+\sqrt{13}}{2}$

9) Solve $\|x^2-5x+1\| = 3$

$x^2-5x+1 = 3 \cup x^2-5x+1 = -3$

$x^2-5x-2 = 0$

$x^2-5x+4 = 0$

$(x-4)(x-1) = 0$

$x = \frac{5\pm\sqrt{25+8}}{2}$

$\boxed{x = \frac{5\pm\sqrt{33}}{2} \cup x = 1, 4}$

Series and Sequence

1) Expand and simplify: $(3a-2b)^5$ $1(3a)^5 - 5(3a)^4(2b) + 10(3a)^3(2b)^2 - 10(3a)^2(2b)^3 + 5(3a)(2b)^4 - 1(2b)^5$

$\boxed{243a^5 - 810a^4b + 1080a^3b^2 - 720a^2b^3 + 240ab^4 - 32b^5}$

Evaluate each : 2) $\sum_{n=7}^{32} 7-3n$ 3) $\sum_{n=1}^{\infty} 2\left(\frac{3}{5}\right)^{n-1}$ 4) $\sum_{k=1}^{11} 3(-2)^{k-1}$

$\frac{(-14+-89)26}{2} = -1339$ $S_\infty = \frac{2}{1-3/5} = 5$ $S = \frac{3(1-(-2)^{11})}{1+2} = 2049$

Logarithms and Exponentials

1.Expand to a single log: A) $\ln\frac{x^2}{(x-1)}$ $2\ln x - \ln(x-1)$ B) $\log_b\frac{\sqrt{x}(y+1)}{z^2}$ $\frac{1}{2}\log_b x + \log_b(y+1) - 2\log_b z$

Solve for x. Round answer to three decimal places.

2. $3^{2x-5} = 2.056$

$\frac{\log 2.056}{\log 3} = 2x-5$

$x = 2.828$

3. $125^{\log_5 2} = x$

$5^{3\log_5 2} = x$

$8 = x$

4. $\left(\frac{1}{2}\right)^{2x} = 8^{x-5}$

$2^{-2x} = 2^{3x-15}$

$x = 3$

5. $\log_{12} x + \log_{12}(x+1) = 1$

$x^2 + x = 12$

$x^2 + x - 12 = 0$

$(x+4)(x-3)$

$x = -4$ ext, 3

6. $e^{2\ln x} = 5$

$x^2 = 5$

$x = \sqrt{5}, -\sqrt{5}$ ext.

7. $\ln\left(\frac{4x+1}{3}\right) = 2$

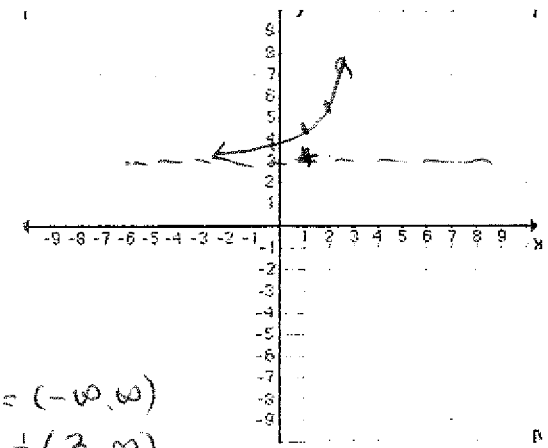
$3e^2 = 4x+1$

$x = 5.292$

Graph

8. $y = 2^{x-1} + 3$

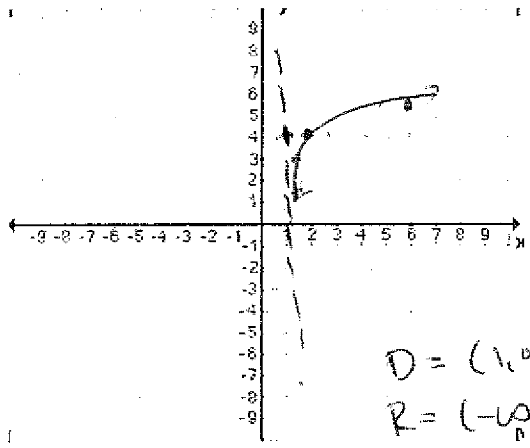
(0, 1)
(1, 2)
(-1, 1/2)



D = $(-\infty, \infty)$
R = $(3, \infty)$
A: $y = 3$

9. $y = \log_5(x-1) + 4$

(1, 0)
(5, 1)
(1/5, -1)



D = $(1, \infty)$
R = $(-\infty, \infty)$
A: $x = 1$

Calculus Concepts

Find the limit of each:

1) $\lim_{x\to 0}\frac{\sqrt{16+x}-4}{x}\left(\frac{\sqrt{16+x}+4}{\sqrt{16+x}+4}\right) = \boxed{\frac{1}{8}}$

2) $\lim_{x\to -1}\frac{x^3-6x^2-x+6}{x+1}$ $\lim_{x\to -1}\frac{\cancel{(x+1)}(x-1)(x-6)}{\cancel{x+1}} = (-2)(-7) = 14$

3) $\lim_{x\to\infty}\frac{4x-7}{\sqrt{x^2+6x}}$ $= 4$

4) $\lim_{x\to\pi}(x\sin x)$

$= \pi\sin\pi$

$= 0$

$\lim_{h\to 0} \frac{3(x+h)^2-6(x+h)+1-3x^2+6x-1}{h}$

$= \lim_{h\to 0} \frac{3x^2+6xh+3h^2-6x-6h+1-3x^2+6x-1}{h}$

$= \lim_{h\to 0} \frac{h(6x+3h-6)}{h}$

5) Find the derivative of the functions using the limit definition. $f(x)=3x^2-6x+1$

$= 6x+0-6$

$f'(x) = 6x-6$

Calculus BC ONLY

1. Find the slope of the tangent line to $f(x)=(x^2+2)^3$ at $x=1$.

$f'(x) = 3(x^2+2)^2(2x)$

$f'(1) = 54$

2. Find the equation of the tangent line to $f(x)=\sqrt{x^2+2x+8}$ at $(2,4)$

$f(x)=(x^2+2x+8)^{1/2}$ $\quad f'(x)=\frac{1}{2}(x^2+2x+8)^{-1/2}(2x+2)$ $\quad f'(2)=(2+1)\frac{1}{4}=\frac{3}{4}$ $\quad \boxed{y-4=\frac{3}{4}(x-2)}$

Find the derivative of the following functions using the shortcut method. Use proper notation!

3. $y=\sqrt[4]{x^3}$

$y=\frac{3}{4}x^{-1/4} = \boxed{\frac{3}{4\sqrt[4]{x}}}$

4. $f(x)=(x+1)(2x-3)$

$2x-3+2(x+1)=f'(x)$

$\boxed{4x-1}$

5. $g(x)=\frac{5}{2x^2}$ $\quad \frac{5x^{-2}}{2}$

$\boxed{g'(x)=\frac{-5}{x^3}}$

6. $h(t)=\frac{t^4+2t^3+3t}{t}$ $= t^3+2t^2+3$

$\boxed{h'(t)=3t^2+4t}$

7. $f(x)=(x^2-2x+1)(x^3-x^2)$

$f'(x)=(2x-2)(x^3-x^2)+(3x^2-2x)(x^2-2x+1)$

$\boxed{= 5x^4-12x^3+9x^2-2x}$

8. $g(x)=\frac{x^3+2}{2x-1}$

$g'(x)=\frac{3x^2(2x-1)-2(x^3+2)}{(2x-1)^2}$

$\boxed{\frac{4x^3-3x^2-4}{(2x-1)^2}}$

9. $g(x)=\sin^2 3x$

$g'(x)= 2\sin(3x)\cdot 3\cos 3x$

$6\sin 3x\cos 3x = \boxed{3\sin 6x}$

10. $f(x)=2\tan 3x$

$f'(x)=2\sec^2(3x)\cdot 3$

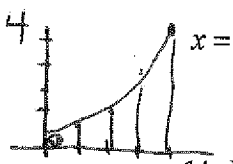
$\boxed{= 6\sec^2(3x)}$

11. $f(x)=x^3\cos x$

$f'(x)=3x^2\cos x+x^3(-\sin x)$

$\boxed{x^2(3\cos x - x\sin x)}$

13. Use 4 rectangles to approximate the area of the region bounded by $f(x)=\frac{1}{4}x^2$, the x-axis, $x=0$ and $x=4$ using left and right Riemann's Sums.



Left $S=1(0+\frac{1}{4}+1+\frac{9}{4})=\frac{7}{2}$

Right $S=1(\frac{1}{4}+1+\frac{9}{4}+4)=\frac{15}{2}$

14. Find k so that f will be continuous at $x=2$, given $f(x)=\begin{cases} kx^2, & x\le 2 \\ 2x+k, & x>2 \end{cases}$

$k(2)^2=2(2)+k$

$4k=4+k$

$3k=4$

$\boxed{k=\frac{4}{3}}$

#15

i) $f(c)$ defined $\forall_c$ $(-\infty,\infty)$

$f(c)=\frac{1}{c^2+4}$ is defined $c^2+4\ne 0$

ii) $\lim_{x\to c}\frac{1}{x^2+4}$ exists? yes

$\lim_{x\to c^-}\frac{1}{x^2+4}=\lim_{x\to c^+}\frac{1}{x^2+4}$

iii) $f(c)=\lim_{x\to c}\frac{1}{x^2+4}$ for $\forall$ i

15. *Discuss the continuity of each function* $f(x)=\frac{1}{x^2+4}$ (3-step process)

Identify the polar graph, then graph noting the length of all loops.

16. $r^2=16\sin(2\theta)$

Lemniscate QI/QIII
4 units each ribbon

17. $r=4\cos(5\theta)$

rose 5 petals
length 4
polar axis

18. $r=3-4\cos\theta$

Limaçon inner loop
outer 7 to left inner 1
polar axis

19. $r=6$

Circle center @ pole
radius = 6

19. $r=1+\sin\theta$

Cardiod up
2 units up

20. $r=5+2\cos\theta$

Limaçon w/o loop
3 units left
7 units right
polar axis