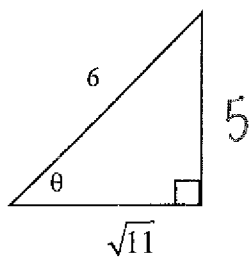
Trigonometry

Give the Exact Value of each without using a calculator.

1) $\tan\left(\frac{5\pi}{6}\right)$ $-\frac{\sqrt{3}}{3}$ 2) $\sin\left(\frac{5\pi}{3}\right)$ $-\frac{\sqrt{3}}{2}$ 3) $\cos 300^o$ $1/2$ 4) $\cot\left(\frac{11\pi}{6}\right)$ $-\sqrt{3}$ 5) $\sec\left(\frac{2\pi}{3}\right)$ -2

6) Find the six trig. ratios for θ.



$\sin\theta = \frac{5}{6}$ $\csc\theta = \frac{6}{5}$

$\cos\theta = \frac{\sqrt{11}}{6}$ $\sec\theta = \frac{6\sqrt{11}}{11}$

$\tan\theta = \frac{5\sqrt{11}}{11}$ $\cot\theta = \frac{\sqrt{11}}{5}$

7) Solve the triangle if $A = 58^o, B = 42^o, and\ c = 104$

$A = 58^o$ $a = 89.558$

$B = 42^o$ $b = 70.663$

$C = 80^o$ $c = 104$

8) A flagpole casts a 60-foot shadow when the angle of elevation of the sun is 35°. Find the height of the flagpole.

$\tan 35 = \frac{x}{60}$ 42.012 ft.

Factoring

Factor each

1) $z^2 - 9z + 14$ $(z-7)(z-2)$

2) $4y^2 - 81$ $(2y+9)(2y-9)$

3) $7z^2 + 23z + 6$ $(z+3)(7z+2)$

4) $4x^2(8y-3)^4 - 2x^3(8y-3)^5$ $2x^2(8y-3)^4(2 - x(8y-3))$

5) $(x-5)^2 + 4(x-5) + 4$ $(x-3)^2$

6) $27y^3 + 64$ $(3y+4)(9y^2 - 12y + 16)$

7) The sides of an acute triangle measures 14 cm, 18 cm, and 20 cm. Which of the following equations, when solved for θ, gives the measure of the smallest angle of the triangle?

[F] $\frac{\sin\theta}{14} = \frac{1}{18}$

[G] $\frac{\sin\theta}{14} = \frac{1}{20}$

[H] $\frac{\sin\theta}{20} = \frac{1}{14}$

[J] $14^2 = 18^2 + 20^2 - 2(18)(20)\cos\theta$

[K] $20^2 = 14^2 + 18^2 - 2(14)(18)\cos\theta$

8) Find the area of the triangle with sides of 9, 10, and 17.

$\sqrt{18 \cdot 9 \cdot 8 \cdot 1} = \boxed{36}$

Functions

1) Write the equation of the line passing through (5, -6) and (12, -1)

$m = \frac{-1+6}{12-5} = \frac{5}{7}$ $y+1 = \frac{5}{7}(x-12)$

or

$y+6 = \frac{5}{7}(x-5)$

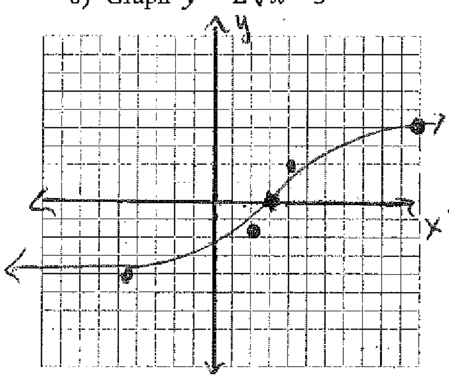
2) Find the distance and midpoint given the points (-2, 1) and (3, 4) $d = \sqrt{5^2+3^2} = \sqrt{34}$ midpt $\left(\frac{1}{2}, \frac{5}{2}\right)$

3) Determine the domain $f(x) = \frac{1}{x^2-4}$ $x \neq 2, -2$ $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$

4) Given $f(x) = x^2 - 2x + 4$, evaluate f(-5) $f(-5) = (-5)^2 - 2(-5) + 4$
$25 + 10 + 4 = 39$

5) Find the inverse: $f(x) = \frac{4}{2x-5}$ $x = \frac{4}{2y-5}$ $2y - 5 = \frac{4}{x}$ $2y = \frac{4}{x} - 5$ $\boxed{f^{-1}(x) = \frac{2}{x} - \frac{5}{2}}$

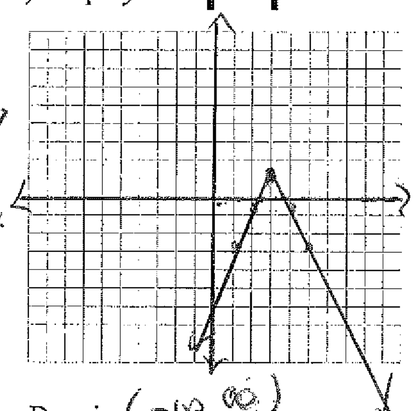
6) Graph $y = 2\sqrt[3]{x-3}$



Domain $(-\infty, \infty)$

Range $(-\infty, \infty)$

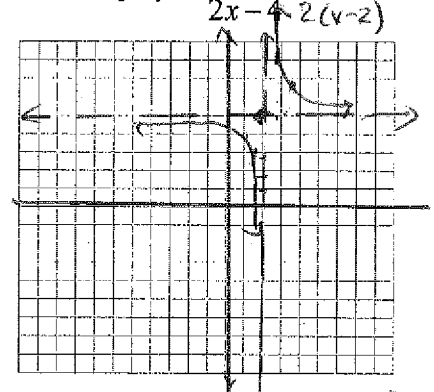
7) Graph $y = -2|x-3|+1$



Domain $(-\infty, \infty)$

Range $(-\infty, 1]$

8) Graph $y = \frac{3}{2x-4} + 5$ $2(x-2)$



Domain $(-\infty, 2) \cup (2, \infty)$

Range $(-\infty, 5) \cup (5, \infty)$

9) Convert from standard form to vertex form: $y = 2x^2 - 8x + 3$ $y = 2(x^2 - 4x + 4) + 3 - 8$
$\boxed{y = 2(x-2)^2 - 5}$

Polynomials

1) Solve $\frac{1}{x-2} = \frac{3}{x+2} - \frac{6x}{x^2-4}$

2) Solve $(3x-1)^{\frac{2}{3} \cdot \frac{3}{2}} = 64^{3/2}$
$3x - 1 = 512$
$\boxed{x = 171 \text{ or } x = \frac{-511}{3}}$

3) $5(x+1)^2 = 60$
$(x+1)^2 = 12$
$(x+1) = \pm 2\sqrt{3}$
$\boxed{x = -1 \pm 2\sqrt{3}}$

4) Simplify each: $(2-8i)-(4+2i)$
$2 - 8i - 4 - 2i$
$\boxed{-2 - 10i}$

$(5+3i)+(13-6i)$
$\boxed{18 - 3i}$

$i^{6,926} = i^2 = -1$

$\frac{1}{5-2i} \cdot \frac{5+2i}{5+2i} = \boxed{\frac{5+2i}{29}}$

$\frac{2+3i}{1+2i} \cdot \frac{1-2i}{1-2i} = \boxed{\frac{8-i}{5}}$

$(3+4i)^2 (3+4i)$
$9 + 12i + 12i + 16i^2$ -16
$\boxed{-7 + 24i}$

5) Divide using long division $\frac{x^3+2x^2-5x-6}{x^2+3x-1}$ $\boxed{x - 1 + \frac{-x-7}{x^2+3x-1}}$

$$
\begin{array}{r}
x - 1 \\
x^2+3x-1 \enclose{longdiv}{x^3 + 2x^2 - 5x - 6} \\
-x^3 - 3x^2 + x \\
\hline
-x^2 - 4x - 6 \\
x^2 + 3x - 1 \\
\hline
-x - 7
\end{array}
$$

6) Divide using synthetic division $\frac{x^4+5x^3-2x-8}{x+3}$

$$
\begin{array}{r|rrrrr}
-3 & 1 & 5 & 0 & -2 & -8 \\
 & \downarrow & -3 & -6 & 18 & \\
\hline
 & 1 & 2 & -6 & 16 & -56
\end{array}
$$

$\boxed{x^3 + 2x^2 - 6x + 16 + \frac{-56}{x+3}}$

$x = \frac{1 \pm \sqrt{1-4(1)(-3)}}{2}$

$\frac{1 \pm \sqrt{13}}{2}$

$- \; 0 \; + \; 0 \; - \; 0 \; +$

$\frac{1-\sqrt{13}}{2} \quad 0 \quad \frac{1+\sqrt{13}}{2}$

Solve each inequality

7) $1 < 5x + 6 < 9$

$-6 \quad -6 \quad -6$

$-5 < 5x < 3$

$\boxed{-1 < x < 3/5}$

8) $x^3 - x^2 - 3x < 0$

$x(x^2 - x - 3) < 0$

$\boxed{(-\infty, \frac{1-\sqrt{13}}{2}) \cup (0, \frac{1+\sqrt{13}}{2})}$

9) $\frac{x-2}{x-1} < 1$

$\frac{x-2}{x-1} - 1 < 0$

$\frac{x-2-x+1}{x-1} < 0$

$\frac{-1}{x-1} < 0$

$+ \; 0 \rightarrow$

1

$\boxed{(1, \infty)}$

10) Solve $|5 + 2j| = 9$

$5 + 2j = 9 \cup 5 + 2j = -9$

$2j = 4 \cup 2j = -14$

$\boxed{j = 2 \cup j = -7}$

11) Solve $|x^2 - 5x + 1| = 3$

$x^2 - 5x + 1 = 3 \cup x^2 - 5x + 1 = -3$

$x^2 - 5x - 2 = 0 \qquad x^2 - 5x + 4 = 0$

$\frac{5 \pm \sqrt{25+8}}{2} \qquad (x-4)(x-1)$

$\boxed{x = 4, 1, \frac{5 \pm \sqrt{33}}{2}}$

Logarithms and Exponentials

1.Condense to a single log: A) $2\ln(4) - \ln(3)$ $\boxed{\ln \frac{16}{3}}$

B) $\frac{1}{2}\log_b x + \log_b(y+1) - 2\log_b z$

$\log_b \left(\frac{\sqrt{x}(y+1)}{z^2} \right)$

2) Expand using Pascal's Triangle $(r - b^2)^6$

$\boxed{r^6 - 6r^5b^2 + 15r^4b^4 - 20r^3b^6 + 15r^2b^8 - 6rb^{10} + b^{12}}$

Solve for x. Round answer to three decimal places.

3. $\log_8 175 = x$

$\frac{\log 175}{\log 8} \approx 2.484$

4. $3^{2x-5} = 2.056$

$\log_3 2.056 = 2x - 5$

$x \approx 2.828$

5. $125^{\log_5 2} = x$

$5^{3\log_5 2} = x$

$2^3 = x$

$x = 8$

6. $\log_2 x^3 = 9$

$3\log_2 x = 9$

$\log_2 x = 3$

$\boxed{x = 8}$

7. $\left(\frac{1}{2}\right)^{2x} = 8^{x-5}$

$2^{-2x} = 2^{3x-15}$

$-5x = -15$

$x = 3$

8. $\log_{12} x + \log_{12}(x+1) = 1$

$\log_{12} x^2 + x = 1$

$x^2 + x = 12$

$x^2 + x - 12 = 0$

$(x+4)(x-3) = 0$

$\boxed{3, -4 \text{ ext.}}$

10. $e^{2\ln x} = 5$

$x^2 = 5$

$\boxed{x = \sqrt{5}, -\sqrt{5} \text{ ext.}}$

11. $\ln\left(\frac{4x+1}{3}\right) = 2$

$3e^2 = 4x + 1$

$\boxed{x \approx 5.292}$

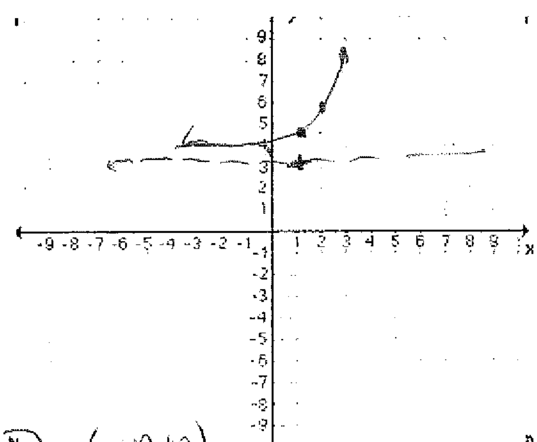
Graph

12. $y = 2^{x-1} + 3$

(0, 1)

(1, 2)

$(-1, \frac{1}{2})$



$D = (-\infty, \infty)$

$R = (3, \infty)$

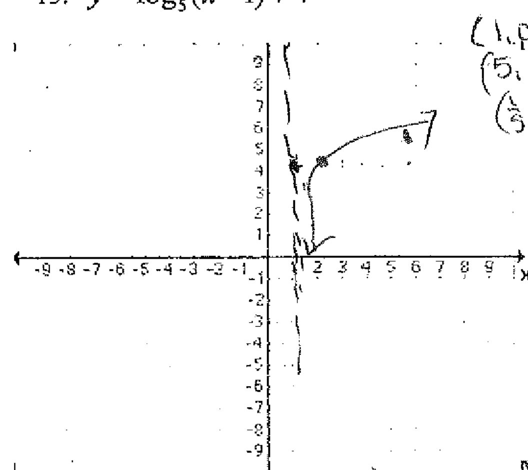
A: $y = 3$

13. $y = \log_5(x-1) + 4$

(1, 0)

(5, 1)

$(\frac{1}{5}, -1)$



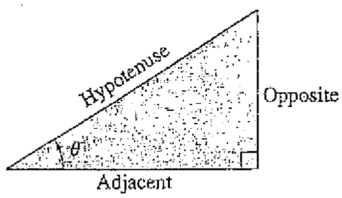
$D = (1, \infty)$

$R = (-\infty, \infty)$

A: $x = 1$

Definition of the Six Trigonometric Functions

Right triangle definitions, where $0 < \theta < \pi/2$.

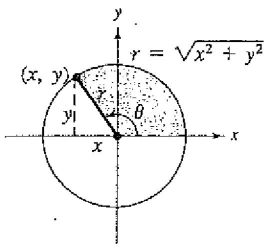


$$\sin\theta = \frac{\text{opp.}}{\text{hyp.}} \quad \csc\theta = \frac{\text{hyp.}}{\text{opp.}}$$

$$\cos\theta = \frac{\text{adj.}}{\text{hyp.}} \quad \sec\theta = \frac{\text{hyp.}}{\text{adj.}}$$

$$\tan\theta = \frac{\text{opp.}}{\text{adj.}} \quad \cot\theta = \frac{\text{adj.}}{\text{opp.}}$$

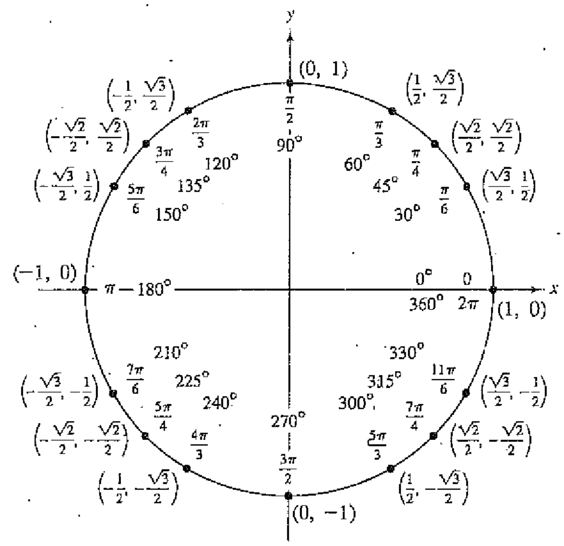
Circular function definitions, where θ is any angle.



$$\sin\theta = \frac{y}{r} \quad \csc\theta = \frac{r}{y}$$

$$\cos\theta = \frac{x}{r} \quad \sec\theta = \frac{r}{x}$$

$$\tan\theta = \frac{y}{x} \quad \cot\theta = \frac{x}{y}$$



Logarithms:

$y = \log_a x$ is equivalent to $x = a^y$

Product property: $\log_b mn = \log_b m + \log_b n$

Quotient property: $\log_b \frac{m}{n} = \log_b m - \log_b n$

Power property: $\log_b m^p = p \log_b m$

Property of equality: If $\log_b m = \log_b n$, then m = n

Change of base formula: $\log_a n = \frac{\log_b n}{\log_b a}$

Fractional exponent: $\sqrt[b]{x^e} = x^{\frac{e}{b}}$

Negative Exponents: $x^{-n} = 1/x^n$

The Zero Exponent: $x^0=1$, for x not equal to 0.

Multiplying Powers

Multiplying Two Powers of the Same Base:
$(x^a)(x^b) = x^{(a+b)}$

Multiplying Powers of Different Bases:
$(xy)^a = (x^a)(y^a)$

Dividing Powers

Dividing Two Powers of the Same Base:
$(x^a)/(x^b) = x^{(a-b)}$

Dividing Powers of Different Bases:
$(x/y)^a = (x^a)/(y^a)$

Slope-intercept form: $y = mx + b$

Point-slope form: $y = m(x - x_1) + y_1$

Standard form: $Ax + By + C = 0$

The circle is known as the **unit circle**, since its radius is one unit. It is an integral part of trigonometry.

You will cut out the triangles on the next page to help fill in the coordinates of this unit circle. Use what you know about reflections to help!

Remember

- * cos = x-value
- * sin = y-value
- * csc → sin
- * sec → cos
- * cot → tan
- * tan → sin/cos

↑ I work down as we can across in notes. Gives 1 page to study

HINTS FOR QUADRANT II:

The angles in quadrant II are 120°, 135°, and 150°.

Look at the 120° point and the 60° point.
What should be true about their *y*-coordinates?

What should be true about their *x*-coordinates?