Law of Detachment Deductive reasoning is the process of using facts, rules, definitions, or properties to reach conclusions. One form of deductive reasoning that draws conclusions from a true conditional $p \rightarrow q$ and a true statement $p$ is called the Law of Detachment.

| Law of Detachment | If $p \rightarrow q$ is true and $p$ is true, then $q$ is true. |

Example Determine whether each conclusion is valid based on the given information. If not, write invalid. Explain your reasoning.

a. Given: Two angles supplementary to the same angle are congruent. $\angle A$ and $\angle C$ are supplementary to $\angle B$.
   
   Conclusion: $\angle A$ is congruent to $\angle C$.
   
   The statement $\angle A$ and $\angle C$ are supplementary to $\angle B$ is the hypothesis of the conditional. Therefore, by the Law of Detachment, the conclusion is true.

b. Given: If Helen is going to work, then she is wearing pearls. Helen is wearing pearls.
   
   Conclusion: Helen is going to work.
   
   The given statement Helen is going to work satisfies the conclusion of the true conditional. However, knowing that a conditional statement and its conclusion are true does not make the hypothesis true. Helen could be wearing pearls on a date. The conclusion is invalid.

Exercises Determine whether the stated conclusion is valid based on the given information. If not, write invalid. Explain your reasoning.

1. Given: If a number is divisible by 6, then the number is divisible by 3.
   
   18 is divisible by 6.
   
   Conclusion: 18 is divisible by 3.

2. Given: If a pet is a rabbit, then it eats carrots. Jennie’s pet eats carrots.
   
   Conclusion: Jennie’s pet is a rabbit.

3. Given: If a hen is a Plymouth Rock, then her eggs are brown.
   
   Berta is a Plymouth Rock hen.
   
   Conclusion: Berta’s eggs are brown.
Law of Syllogism  Another way to make a valid conclusion is to use the Law of Syllogism. It allows you to draw conclusions from two true statements when the conclusion of one statement is the hypothesis of another.

The two conditional statements below are true. Use the Law of Syllogism to find a valid conclusion. State the conclusion.

(1) If a number is a whole number, then the number is an integer.
(2) If a number is an integer, then it is a rational number.

\[ p: \text{A number is a whole number.} \]
\[ q: \text{A number is an integer.} \]
\[ r: \text{A number is a rational number.} \]

The two conditional statements are \( p \to q \) and \( q \to r \). Using the Law of Syllogism, a valid conclusion is \( p \to r \). A statement of \( p \to r \) is “if a number is a whole number, then it is a rational number.”

Exercises

Use the Law of Syllogism to draw a valid conclusion from each set of statements, if possible. If no valid conclusion is possible, write no valid conclusion.

1. If a dog eats Superdog Dog Food, he will be happy.
   Rover is happy.

2. If an angle is supplementary to an obtuse angle, then it is acute.
   If an angle is acute, then its measure is less than 90.

3. If the measure of \( \angle A \) is less than 90, then \( \angle A \) is acute.
   If \( \angle A \) is acute, then \( \angle A \cong \angle B \).

4. If an angle is a right angle, then the measure of the angle is 90.
   If two lines are perpendicular, then they form a right angle.

5. If you study for the test, then you will receive a high grade.
   Your grade on the test is high.
2-4 Skills Practice

Deductive Reasoning

Determine whether the stated conclusion is valid based on the given information. If not, write invalid. Explain your reasoning.

1. Given: If the sum of the measures of two angles is 180, then the angles are supplementary. \( m\angle A + m\angle B \) is 180.
   Conclusion: \( \angle A \) and \( \angle B \) are supplementary.

2. Given: If the sum of the measures of two angles is 90, then the angles are complementary. \( m\angle ABC \) is 45 and \( m\angle DEF \) is 48.
   Conclusion: \( \angle ABC \) and \( \angle DEF \) are complementary.

3. Given: If the sum of the measures of two angles is 180, then the angles are supplementary. \( \angle 1 \) and \( \angle 2 \) are a linear pair.
   Conclusion: \( \angle 1 \) and \( \angle 2 \) are supplementary.

Use the Law of Syllogism to draw a valid conclusion from each set of statements, if possible. If no valid conclusion can be drawn write no valid conclusion and explain your reasoning.

4. If two angles are complementary, then the sum of their measures is 90.
   If the sum of the measures of two angles is 90, then both of the angles are acute.

5. If the heat wave continues, then air conditioning will be used more frequently.
   If air conditioning is used more frequently, then energy costs will be higher.

6. If it is Tuesday, then Marla tutors chemistry.
   If Marla tutors chemistry, then she arrives home at 4 P.M.

7. If a marine animal is a starfish, then it lives in the intertidal zone of the ocean.
   The intertidal zone is the least stable of the ocean zones.
2-4 Practice

Deductive Reasoning

Determine whether the stated conclusion is valid based on the given information. If not, write invalid. Explain your reasoning.

1. Given: If a point is the midpoint of a segment, then it divides the segment into two congruent segments. $R$ is the midpoint of $QS$.
   Conclusion: $QR \cong RS$.

2. Given: If a point is the midpoint of a segment, then it divides the segment into two congruent segments. $AB \cong BC$.
   Conclusion: $B$ divides $AC$ into two congruent segments.

Use the Law of Syllogism to draw a valid conclusion from each set of statements, if possible. If no valid conclusion can be drawn, write no valid conclusion.

3. If two angles form a linear pair, then the two angles are supplementary.
   If two angles are supplementary, then the sum of their measures is 180.

4. If a hurricane is Category 5, then winds are greater than 155 miles per hour.
   If winds are greater than 155 miles per hour, then trees, shrubs, and signs are blown down.

Draw a valid conclusion from the statements, if possible. Then state whether your conclusion was drawn using the Law of Detachment or the Law of Syllogism. If no valid conclusion can be drawn, write no valid conclusion and explain your reasoning.

5. Given: If a whole number is even, then its square is divisible by 4.
   The number I am thinking of is an even number.

6. BIOLOGY If an organism is a parasite, then it survives by living on or in a host organism. If a parasite lives in or on a host organism, then it harms its host. What conclusion can you draw if a virus is a parasite?