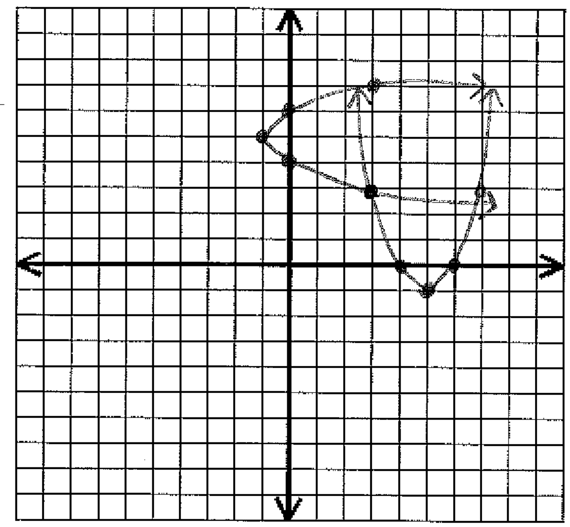
Algebra II Unit 9 Analyze Quadratics and their Inverses **NOTES 9-1**

Graph the function: $f(x) = (x - 5)^2 - 1$

*What point do we already know from this form? (5, -1)

*What else does this form tell us? opens up



x	Substitute x value to determine y	y
3	$(3-5)^2-1 = (-2)^2-1 = 4-1 = 3$	3
4	$(4-5)^2-1 = (-1)^2-1 = 1-1 = 0$	0
5		-1
6	$(6-5)^2-1 = (1)^2-1 = 1-1 = 0$	0
7	$(7-5)^2-1 = (2)^2-1 = 4-1 = 3$	3

Domain: $\mathbb{R}$ Range: $[-1, \infty)$

Is this a function? Yes

Inverse: notation is written as $f^{-1}(x)$; the input (x) and output (y) switch places → then solve for y to determine the inverse equation

$f^{-1}(x)$

x	y
3	3
0	4
-1	5
0	6
3	7

Domain: $[-1, \infty)$ Range: $\mathbb{R}$

Is the inverse a function? No

$\overset{y}{\cancel{f(x)}} = (x-5)^2 - 1$

$x = (y-5)^2 - 1$

$\;\;+1 \qquad\qquad +1$

$x + 1 = (y-5)^2$

$\pm\sqrt{x+1} = y - 5$

$\;\;+5 \qquad +5$

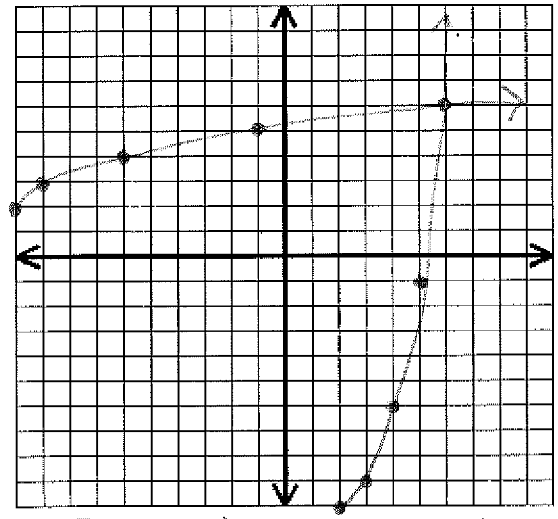
$\pm\sqrt{x+1} + 5 = y$

Inverse equation: $f^{-1}(x) = \pm\sqrt{x+1} + 5$

Graph the function: $f(x)=\sqrt{x+10}+2$

*What point do we already know from this form? $(-10, 2)$

x	Substitute x value to determine y	y
-10		2
-9	$\sqrt{-9+10}+2=\sqrt{1}+2=1+2=3$	3
-6	$\sqrt{-6+10}+2=\sqrt{4}+2=2+2=4$	4
-1	$\sqrt{-1+10}+2=\sqrt{9}+2=3+2=5$	5
6	$\sqrt{6+10}+2=\sqrt{16}+2=4+2=6$	6



Domain: $[-10, \infty)$ Range: $[2, \infty)$

Is this a function? Yes

Inverse: notation is written as $f^{-1}(x)$; the input (x) and output (y) switch places → then solve for y to determine the inverse equation

$f^{-1}(x)$

x	y
2	-10
3	-9
4	-6
5	-1
6	6

Domain: $[2, \infty)$ Range: $[-10, \infty)$

Is the inverse a function? Yes

$y = \cancel{f(x)}=\sqrt{x+10}+2$

$x=\sqrt{y+10}+2$
$-2 \qquad -2$

$x-2=\sqrt{y+10}$

$(x-2)^2=y+10$
$-10 \qquad -10$

$(x-2)^2-10=y$

Inverse equation: $f^{-1}(x)=(x-2)^2-10$

Find the inverse equations of the following:

1) $g(x)=2(x+7)^2-8$

$x=2(y+7)^2-8$
$+8 \qquad +8$

$\frac{1}{2}(x+8)=\frac{2(y+7)^2}{2}$

$\frac{1}{2}(x+8)=(y+7)^2$

$\pm\sqrt{\frac{1}{2}(x+8)}=y+7$
$-7 \qquad -7$

$\pm\sqrt{\frac{1}{2}(x+8)}-7=y$

$g^{-1}(x)=\pm\sqrt{\frac{1}{2}(x+8)}-7$

2) $h(x)=-\sqrt{x-4}+2$

$x=-\sqrt{y-4}+2$
$-2 \qquad -2$

$-1(x-2)=\frac{-\sqrt{y-4}}{-1}$

$[-(x-2)]^2=(\sqrt{y-4})^2$

$[-(x-2)]^2=y-4$
$+4 \qquad +4$

$[-(x-2)]^2+4=y$

$h^{-1}(x)=[-(x-2)]^2+4$

3) $j(x)=\frac{1}{3}(x-8)^2+1$

$x=\frac{1}{3}(y-8)^2+1$
$-1 \qquad -1$

$3(x-1)=\frac{1}{3}(y-8)^2\cdot 3$

$\sqrt{3(x-1)}=\sqrt{(y-8)^2}$

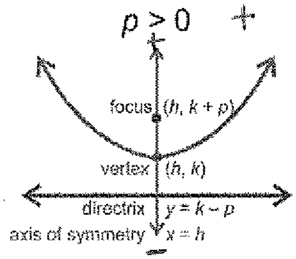
$\pm\sqrt{3(x-1)}=y-8$
$+8 \qquad +8$

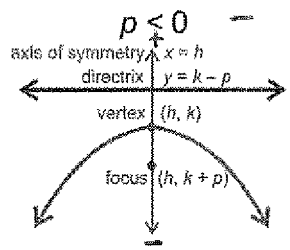
$\pm\sqrt{3(x-1)}+8=y$

$j^{-1}(x)=\pm\sqrt{3(x-1)}+8$

INTRO TO CONIC FORM:

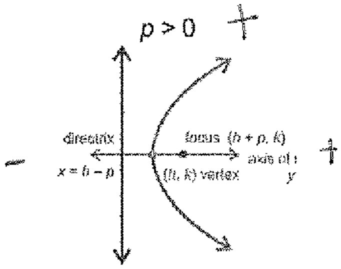
Conic form for a parabola with vertex (h, k) is
$(x - h)^2 = 4p(y - k)$ *vertical axis* Opens up or down.

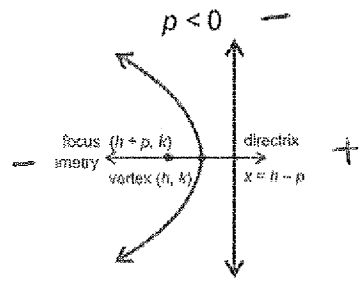




x^2 vs y^2
↓ Up or Down (what we are used to seeing)
↓ Right or Left

$(y - k)^2 = 4p(x - h)$ *horizontal axis* Opens right or left.





Note: p is positive opens toward the positive (up or right)
p is negative opens toward the negative (down or left)
p = the distance from the vertex to the focus and from the vertex to the directrix

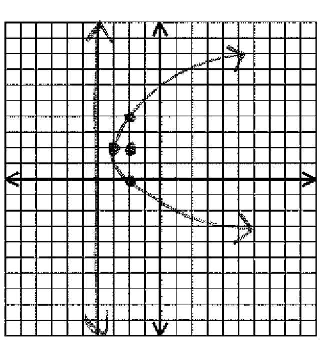
The vertex is halfway between the focus and the directrix.

Draw the focus as a point. Draw the directrix as a line.
Focal Width is a total of 4p units; 2p units on either side of the focus.

Find the vertex, focus, and directrix of each parabola. Then graph each one.

1. $(y - 2)^2 = \overset{4(1)}{4}(x + 3)$

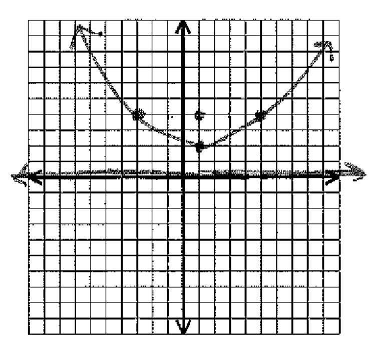
horizontal axis, p +
Opens right



p = 1
Vertex (-3,2)
Focus (-2,2)
Directrix x = -4
EoFW (2p): (-2,4) (-2,0)

2. $(x - 1)^2 = \overset{4(2)}{8}(y - 2)$

vertical axis, p +
Opens up



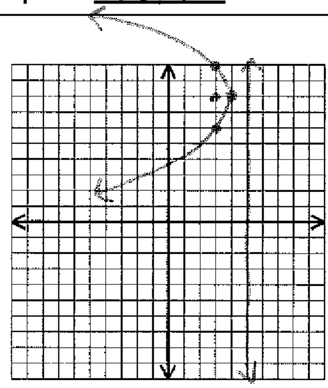
p = 2
Vertex (1,2)
Focus (1,4)
Directrix y = 0
EoFW (2p): (-3,4) (5,4)

4(-1)

3. $(y-8)^2 = -4(x-4)$

horizontal axis, p –

Opens left



p = -1

Vertex (4, 8)

Focus (3, 8)

Directrix x = 5

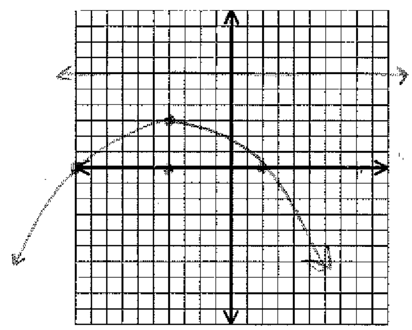
EoFW (2p):
(3, 10) (3, 6)

4(-3)

4. $(x+4)^2 = -12(y-3)$

vertical axis, p –

Opens down



p = -3

Vertex (-4, 3)

Focus (-4, 0)

Directrix y = 6

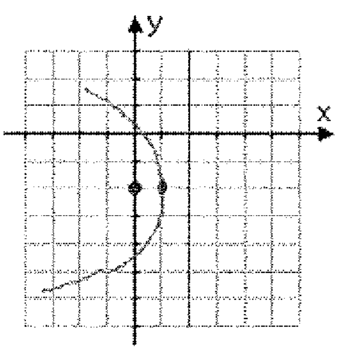
EoFW (2p):
(-10, 0) (2, 0)

Writing a conic form equation from a graph:

1. Choose the correct form of the conic equation. (x^2: opens up/down, y^2: opens right/left)
2. Find p. Is it positive or negative?
3. Find the coordinates of the vertex. (h goes with x, k goes with y)

1. $(y+2)^2 = 4(-1)(x-1)$

$(y+2)^2 = -4(x-1)$



p = -1

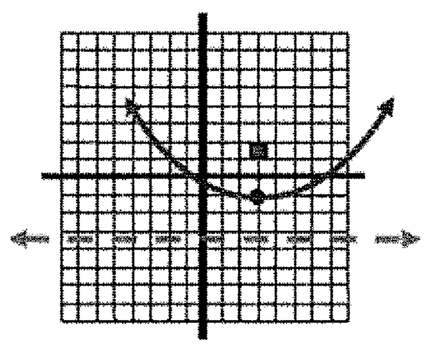
Vertex (1, -2)

Focus (0, -2)

Directrix x = 2

2. $(x-3)^2 = 4(2)(y+1)$

$(x-3)^2 = 8(y+1)$



p = 2

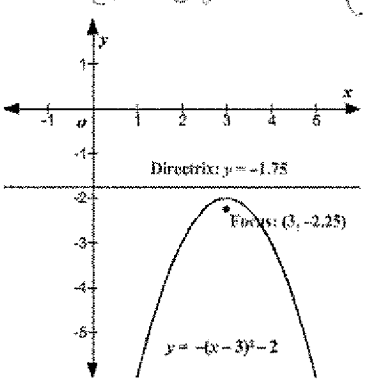
Vertex (3, -1)

Focus (3, 1)

Directrix y = -3

3. $(x-3)^2 = 4(-.25)(y+2)$

$(x-3)^2 = -(y+2)$



p = -.25

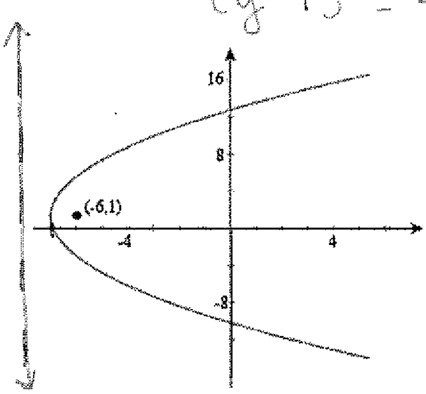
Vertex (3, -2)

Focus (3, -2.25)

Directrix y = -1.75

4. $(y-1)^2 = 4(1)(x+7)$

$(y-1)^2 = 4(x+7)$



p = 1

Vertex (-7, 1)

Focus (-6, 1)

Directrix x = -8

2nd Matrix → Edit A
2nd Matrix → Edit B
2nd Quit
2nd Matrix → A → x^{-1}
2nd Matrix → B → Enter

Write the equation of a parabola $f(x) = ax^2 + bx + c$ Given 3 points on the curve.

1. (3,20) ; (-1, -4) ; (-5, 4)

Substitute each x and y into the equation.

$a(3)^2 + b(3) + c \rightarrow 9a + 3b + c = 20$

$a(-1)^2 + b(-1) + c \rightarrow 1a - 1b + c = -4$

$a(-5)^2 + b(-5) + c \rightarrow 25a - 5b + c = 4$

$$\begin{bmatrix} 9 & 3 & 1 \\ 1 & -1 & 1 \\ 25 & -5 & 1 \end{bmatrix} \begin{bmatrix} 20 \\ -4 \\ 4 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ -1 \end{bmatrix}$$

Solve the system using matrices either A^{-1} B or RREF. a= 1 b= 4 c= -1

Write the equation in standard form $y = 1x^2 + 4x - 1$ y intercept (0, -1)

$(x+2)^2 - 1 - 4$

Write the equation in vertex form $y = (x+2)^2 - 5$ AOS $x = -2$ vertex $(-2, -5)$

2. (9, -3) ; (6, 3) ; (4, 27)

Substitute each x and y into the equation.

$a(9)^2 + b(9) + c \rightarrow 81a + 9b + c = -3$

$a(6)^2 + b(6) + c \rightarrow 36a + 6b + c = 3$

$a(4)^2 + b(4) + c \rightarrow 16a + 4b + c = 27$

$$\begin{bmatrix} 81 & 9 & 1 \\ 36 & 6 & 1 \\ 16 & 4 & 1 \end{bmatrix} \begin{bmatrix} -3 \\ 3 \\ 27 \end{bmatrix} = \begin{bmatrix} 2 \\ -32 \\ 123 \end{bmatrix}$$

Solve the system using matrices either A^{-1} B or RREF. a= 2 b= -32 c= 123

Write the equation in standard form $y = 2x^2 - 32x + 123$ y intercept (0, 123)

Write the equation of a parabola $f(x) = ax^2 + bx + c$. Given 3 points on the curve.

3. (2,-3) ; (-1, 6) ; (4, 1)

Substitute each x and y into the equation.

$a(2)^2 + b(2) + c \rightarrow 4a + 2b + c = -3$

$a(-1)^2 + b(-1) + c \rightarrow 1a - 1b + c = 6$

$a(4)^2 + b(4) + c \rightarrow 16a + 4b + c = 1$

$$\begin{bmatrix} 4 & 2 & 1 \\ 1 & -1 & 1 \\ 16 & 4 & 1 \end{bmatrix} \begin{bmatrix} -3 \\ 6 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -4 \\ 1 \end{bmatrix}$$

Solve the system using matrices either A^{-1} B or RREF. a= 1 b= -4 c= 1

Write the equation in standard form $y = 1x^2 - 4x + 1$ y intercept (0, 1)

$(x-2)^2 + 1 - 4$

$(x-2)^2 - 3$

Write the equation in vertex form $y = (x-2)^2 - 3$ AOS $x = 2$ vertex (2 , -3)

4. (-1, -9) ; (0, -11) ; (-2, -13)

Substitute each x and y into the equation.

$a(-1)^2 + b(-1) + c \rightarrow 1a - 1b + c = -9$

$a(0)^2 + b(0) + c \rightarrow 0a + 0b + c = -11$

$a(-2)^2 + b(-2) + c \rightarrow 4a - 2b + c = -13$

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 0 & 1 \\ 4 & -2 & 1 \end{bmatrix} \begin{bmatrix} -9 \\ -11 \\ -13 \end{bmatrix} = \begin{bmatrix} -3 \\ -5 \\ -11 \end{bmatrix}$$

Solve the system using matrices either A^{-1} B or RREF. a= -3 b= -5 c= -11

Write the equation in standard form $y = -3x^2 - 5x - 11$ y intercept (0, -11)