NOTES 6-1

ALGEBRA II UNIT 6 Solving Quadratics

What is a GCF??

→ The largest number and/or variable that all factors of your expression have in common

How to find the GCF…

Find the GCF of **15** and **30**

factors of **15** (1 , 3 , 5 , 15)   factors of **30** (1 , 2 , 3 , 5 , 6 , 10 , 15 , 30)

→ the largest number they have in common is 5 making it the GCF

Find the GCF of two algebraic terms: $\mathbf{14x^2y^3}$ and **21x**

1. Determine the numeric coefficient of each term (**14** and **21**) and find the GCF:

   factors of **14** (1 , 2 , 7 , 14)   factors of **21** (1 , 3 , 7 , 21)

   → the two coefficients have a 7 in common making it the GCF of the coefficients

2. Find the GCF of the variables of the two terms:

   $\mathbf{14x^2y^3} = x \cdot x \cdot y \cdot y \cdot y$ $\mathbf{21x} = x$

   → The two terms have one variable in common *x*, making the GCF of the variables *x*

3. The GCF of the coefficients times the GCF of the variables is the GCF of the two terms.

   →Answer: $\mathbf{14x^2y^3}$ and **21x** have a GCF of $7x$

Find the GCF of the terms:

1. 60 and 75

   1·60   1·75
   2·30   3·25
   3·20   5·15
   4·15
   5·12
   6·10

   $\boxed{15}$

2. $10xy$ and $25xy$

   1·10   1·25
   2·5   5·5
   x   x
   y   y

   $\boxed{5xy}$

3. $24x^2y^3$ and $36x^2y$

   1·24   1·36
   2·12   2·18
   3·8   3·12
   4·6   4·9
   x·x   6·6
   y·y·y   x·x
       y

   $6x^2y$

OYO - Find the GCF of the terms:

4. a^3b and ab

5. m^4n^5 and n^7

6. xyz^2 and x^2yz

How to Factor out the GCF of the expression: $3x^3 + 27x^2 + 9x$

1. Find the GCF of all of the terms in the expression.

A. Coefficients

3 (1 , 3)
27 (1 , 3 , 9 , 27) → GCF of the coefficients is 3
9 (1 , 3 , 9)

B. Variables

$x^3 =$ x • x • x
$x^2 =$ x • x → GCF of the variables is x
$x =$ x

C. Coefficients and variables written together → **GCF =** 3x

2. Write the GCF on the **left/ outside** of a set of parentheses: 3x (   )

3. Divide each term from the original expression ($3x^3 + 27x^2 + 9x$) by the GCF ($3x$)

$\frac{3x^3}{3x} = 1x^2$ $\frac{27x^2}{3x} = 9x$ $\frac{9x}{3x} = 3$

→These values go inside of the parenthesis: Factored Answer **3x (** $x^2 + 9x + 3$ **)**

You can always check your answer by distributing the GCF back into the parenthesis

Factor out the GCF of the expression.

1. $\frac{5a^2}{5a} + \frac{25a}{5a}$

$5a(a+5)$

2. $\frac{4x^2y^2}{4xy} - \frac{12xy}{4xy}$

$4xy(xy-3)$

3. $\frac{9x^3y^2}{9x^3y^2} + \frac{27x^5y^2}{9x^3y^2} - \frac{81x^6y^2}{9x^3y^2}$

$9x^3y^2(1 + 3x^2 - 9x^3)$

4. $\frac{3x^2}{1} - \frac{4y^2}{1}$

OYO – Factor out the GCF of the expression

5. $x^3 + x^2 + x$

6. 15a + 12b + 6c

7. $15x^2 + -50x + -10$

Factor by Grouping: when there are 4 terms and they all do not share a GCF

$$3x^3 - x^2y + 12x - 4y$$

1. Group the first 2 terms and the last 2 terms together

$\underline{3x^3 - x^2y}$ $\underline{+12x - 4y}$

x^2 4

2. Factor out the GCF from each group.

$\underline{x^2(3x-y) + 4(3x-y)}$

$(3x-y)$

3. Factor out the common binomial

$(3x-y)(x^2+4)$

Factor by grouping.

1. $\underline{x^3+7x^2}$ $\underline{+2x+14}$

x^2 2

$x^2(x+7)+2(x+7)$

$(x+7)(x^2+2)$

2. $\underline{4x^3+2x^2}$ $\underline{-2x-1}$

$2x^2$ -1

$2x^2(2x+1)-1(2x+1)$

$(2x+1)(2x^2-1)$

3. $\underline{2x^3+10x^2}$ $\underline{-3x-15}$

$2x^2$ -3

$2x^2(x+5)-3(x+5)$

$(x+5)(2x^2-3)$

OYO

4. $9x^3+36x^2-4x-16$

NOTES 6-2

ALGEBRA II UNIT 6 Solving Quadratics

Factoring and Solving Trinomials:

Standard Form of a Quadratic Equation: $ax^2 + bx + c = 0$

1. Make sure the equation is equal to 0.
2. ***Factor out a GCF if there is one.***
3. Find the target product by multiplying $a \bullet c$
4. Find the target sum b
5. Split the middle term into 2 new terms (The 2 terms that multiply to give $a \bullet c$ and add up to b)
6. Factor the 4 terms by grouping or using the BOX
7. Set each binomial equal to zero and solve for the variable.

Examples:

1. $3x^2 + 7x + 6 = 4$

$-4 \quad -4$

$3x^2 + 7x + 2 = 0$

$3 \cdot 2$	7
TP	TS
$6 \cdot 1$	$6+1$

$\underline{3x^2 + 6x}\ \underline{+1x + 2} = 0$

$3x \qquad 1$

$3x(x+2) + 1(x+2) = 0$

$(x+2)(3x+1) = 0$

$x + 2 = 0$ $\qquad$ $3x + 1 = 0$

$-2 \quad -2$ $\qquad$ $-1 \quad -1$

$\boxed{x = -2}$ $\qquad$ $\frac{3x}{3} = \frac{-1}{3}$

$x = -1/3$

$\{-2, 1/3\}$

2. $2x^2 + 5x - 2 = -5$

$+5 \quad +5$

$2x^2 + 5x + 3 = 0$

6	5
TP	TS
$2 \cdot 3$	$2+3$

$\underline{2x^2 + 2x}\ \underline{+3x + 3} = 0$

$2x \qquad 3$

$2x(x+1) + 3(x+1) = 0$

$(x+1)(2x+3) = 0$

$x + 1 = 0$ $\qquad$ $2x + 3 = 0$

$-1 \quad -1$ $\qquad$ $-3 \quad -3$

$x = -1$ $\qquad$ $\frac{2x}{2} = \frac{-3}{2}$

$x = -3/2$

$\{-1, -3/2\}$

2. $3x^2 + 10x - 63 = -2x$

$+2x \qquad +2x$

$\frac{3x^2 + 12x - 63 = 0}{3}$

-21	4
TP	TS
$-3 \cdot 7$	$-3+7$

$\frac{\cancel{3}(x^2 + 4x - 21)}{\cancel{3}} = \frac{0}{3}$

$x^2 + 4x - 21 = 0$

$\underline{x^2 - 3x}\ \underline{+7x - 21} = 0$

$x \qquad 7$

$x(x-3) + 7(x-3) = 0$

$(x-3)(x+7) = 0$

$x - 3 = 0$ $\qquad$ $x + 7 = 0$

$x = 3$ $\qquad$ $x = -7$

$\{3, -7\}$

4. $6x^2 - 18x - 18 = 6$

$-6 \quad -6$

$\frac{6x^2 - 18x - 24 = 0}{6}$

-4	-3
TP	TS
$-4 \cdot 1$	$-4+1$

$\frac{\cancel{6}(x^2 - 3x - 4)}{\cancel{6}} = \frac{0}{6}$

$x^2 - 3x - 4 = 0$

$\underline{x^2 - 4x}\ \underline{+1x - 4} = 0$

$x \qquad 1$

$x(x-4) + 1(x-4) = 0$

$(x-4)(x+1) = 0$

$x - 4 = 0$ $\qquad$ $x + 1 = 0$

$x = 4$ $\qquad$ $x = -1$

$\{4, -1\}$

SPECIAL CASE

5. $2x^2 - 32 = 0$

$\frac{2x^2-32}{2}$

$\frac{2(x^2-16)}{2} = \frac{0}{2}$

-16	0
TP	TS
-4·4	-4+4

$(x^2-16)=0$

$\underbrace{x^2-4x}_{x} + \underbrace{4x-16}_{4} = 0$

$x(x-4)+4(x-4)=0$

$(x-4)(x+4)=0$

$x-4=0 \quad x+4=0$

$+4 \;\; +4 \quad -4 \;\; -4$

$\{4,-4\} \quad x=4 \quad x=-4$

6. $5x^2 - 45 = 0$

$\frac{5x^2-45}{5}$

-9	0
TP	TS
-3·3	-3+3

$\frac{5(x^2-9)}{5} = \frac{0}{5}$

$(x^2-9)=0$

$(x+3)(x-3)=0$

$x+3=0 \quad x-3=0$

$x=-3 \quad x=3$

$\{-3, 3\}$

8. $8x^2 - 24x = 0$

$\frac{8x^2-24x}{8x}$

$8x(x-3)=0$

$\frac{8x}{8} = \frac{0}{8} \quad x-3=0$

$+3 \;\; +3$

$x=0 \quad x=3$

$\{0, 3\}$

9. $5x^3 + 25x^2 = 0$

$\frac{5x^3+25x^2}{5x^2}$

$5x^2(x+5)=0$

$\frac{5x^2}{5} = \frac{0}{5} \quad x+5=0$

$-5 \;\; -5$

$\sqrt{x^2} \; \sqrt{0} \quad x=-5$

$x=0 \quad \{0,-5\}$

ALGEBRA II UNIT 6 Solving Quadratics

STANDARD FORM (y-intercept)	VERTEX FORM (vertex)
$0 = ax^2 + bx + c$	$0 = a(x - h)^2 + k$

Solve by completing the square:

Step 1: Place parenthesis around the *a* and *b* terms of the trinomial	Ex. $1(x^2 + 6x) - 11 = 0$
Step 2: Use the formula $(\frac{b}{2})^2$ to find what needs to be added to the parenthesis (to complete it, making it a perfect square trinomial)	$\left(\frac{6}{2}\right)^2 = (3)^2 = 9$
Step 3: Add that value to the parenthesis AND subtract from the end of the problem (to keep the equation balanced)	$(x^2 + 6x + \underline{9}) \underbrace{-11 - \underline{9}}_{-20} = 0$
Step 4: Factor the expression inside the parenthesis and write as a binomial squared. Combine terms outside of the parenthesis. *This is VERTEX form.*	TP 9 | TS 6 $3 \cdot 3$ | $3+3$ $x^2 + 6x + 9$ $x^2 + 3x + 3x + 9$ $x \quad\quad 3$ $x(x+3) + 3(x+3)$ $(x+3)(x+3)$ $(x+3)^2 - 20 = 0$
Step 5: Solve for x. * If we put in a $\sqrt{\ }$ we must also include $\pm$ $\pm\sqrt{\ }$	$(x+3)^2 - 20 = 0$ $+20 \quad +20$ $\sqrt{(x+3)^2} = \sqrt{20}$ $x + 3 = \pm\sqrt{20}$ $x + 3 = \pm 2\sqrt{5}$ $-3 \quad -3$ $x = -3 \pm 2\sqrt{5}$ $\sqrt{20}$ 2 10 2 5 $2\sqrt{5}$

Step 1: Place parenthesis around the *a* and *b* terms of the trinomial	Ex. $2(x^2 + 16x) + 32 = 0$
Step 2: Use the formula $(\frac{b}{2})^2$ to find what needs to be added to the parenthesis (to complete it, making it a perfect square trinomial)	$\left(\frac{16}{2}\right)^2 = (8)^2 = 64$
Step 3: Add that value to the parenthesis AND subtract from the end of the problem (to keep the equation balanced)	$(x^2 + 16x + \underline{64}) \underbrace{+32 - \underline{64}}_{-32} = 0$ 64 16 TP | TS $8 \cdot 8$ | $8+8$
Step 4: Factor the expression inside the parenthesis and write as a binomial squared. Combine terms outside of the parenthesis. *This is VERTEX form.*	$(x+8)(x+8) - 32 = 0$ $(x+8)^2 - 32 = 0$
Step 5: Solve for x.	$(x+8)^2 - 32 = 0$ $+32 \quad +32$ $\sqrt{(x+8)^2} = \sqrt{32}$ $x+8 = \pm\sqrt{32}$ $x+8 = \pm 4\sqrt{2}$ $-8 \quad -8$ $x = -8 \pm 4\sqrt{2}$ $\sqrt{32}$ 2 16 2 8 2 4 2 2 $2 \cdot 2\sqrt{2}$ $4\sqrt{2}$

Complete the square to get the equation in vertex form, then solve for x.

1. $x^2-12x-24=0$

$$\left(\frac{-12}{2}\right)^2=(-6)^2=36$$

$$(x^2-12x\underline{+36})-24-\underline{36}=0$$

$$(x-6)^2-60=0$$

$$+60 \quad +60$$

$$\sqrt{(x-6)^2}=\sqrt{60}$$

$$x-6=\pm\sqrt{60}$$

$$x-6=\pm 2\sqrt{15}$$

$$+6 \quad +6$$

$$x=6\pm 2\sqrt{15}$$

$\sqrt{60}$
2 30
2 15
3 5
$2\sqrt{15}$

2. $x^2+12x+16=0$

$$\left(\frac{12}{2}\right)^2=(6)^2=36$$

$$(x^2+12x+\underline{36})+16-\underline{36}=0$$

$$(x+6)^2-20=0$$

$$(x+6)^2=20$$

$$x+6=\pm\sqrt{20}$$

$$x+6=\pm 2\sqrt{5}$$

$$-6 \quad -6$$

$$x=-6\pm 2\sqrt{5}$$

$\sqrt{20}$
2 10
2 5

3. $x^2-10x+4=0$

$$\left(-\frac{10}{2}\right)^2=(-5)^2=25$$

$$(x^2-10x+\underline{25})+4-\underline{25}=0$$

$$(x-5)^2-21=0$$

$$(x-5)^2=21$$

$$x-5=\pm\sqrt{21}$$

$$+5 \quad +5$$

$$x=5\pm\sqrt{21}$$

$\sqrt{21}$
3 7

4. $x^2+8x-14=0$

$$\left(\frac{8}{2}\right)^2=(4)^2=16$$

$$(x^2+8x+16)-14-16=0$$

$$(x+4)^2-30=0$$

$$(x+4)^2=30$$

$$x+4=\pm\sqrt{30}$$

$$-4 \quad -4$$

$$x=-4\pm\sqrt{30}$$

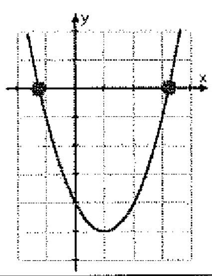
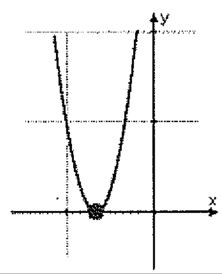
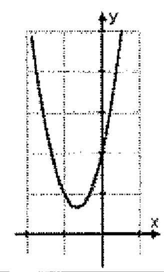
$\sqrt{30}$
2 15
3 5

NOTES 6-4

ALGEBRA II UNIT 6 SOLVING QUADRATICS

The <u>**DISCRIMINANT**</u> determines how many solutions a quadratic equation will have and whether they are real or imaginary.

Discriminant: $b^2 - 4ac$

$b^2 - 4ac > 0$	$b^2 - 4ac = 0$	$b^2 - 4ac < 0$
Two Real Solutions Two x-intercepts	One Real Solution One x-intercept	No Real Solution Two Imaginary Solutions
		

$y = x^2 - 6x + 11$

a = 1
b = -6
c = 11

$(-6)^2 - 4(1)(11)$

$36 - 44$

-8

Negative →

2 imaginary solutions

$f(x) = 2x^2 - 3x - 5$

a = 2
b = -3
c = -5

$(-3)^2 - 4(2)(-5)$

$9 + 40$

49

Positive →

2 real solutions

$-4x^2 + 12x - 9$

$\cancel{x^2 + 6x + 9 = 0}$

a = -4
b = 12
c = -9

$(12)^2 - 4(-4)(-9)$

$144 - 144$

0

Zero →

1 real solution

Rationalize the radicals.

$\sqrt{32}$ $\quad$ $\sqrt{49}$ $\quad$ $-\sqrt{12}$ $\quad$ $\sqrt{-12}$ $\quad$ $5\sqrt{-121}$

$2 \cdot 2\sqrt{2}$

$4\sqrt{2}$

7

$-2\sqrt{3}$

$2i\sqrt{3}$

$5 \cdot 11i$

$55i$

When a quadratic equation is not factorable, use the **QUADRATIC FORMULA** to find your roots/zeros/x-intercepts/solutions.

$$\text{Quadratic Formula: } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

discriminant

1. Make sure the equation is equal to 0 before beginning.
2. When simplifying, enter just what is under the radical into the calculator.
3. The rest needs to be simplified by hand, keeping an exact answer.

$$x^2 + 3x - 10 = 0$$

a = 1

b = 3

c = -10

$$x = \frac{-3 \pm \sqrt{(3)^2 - 4(1)(-10)}}{2(1)}$$

$$x = \frac{-3 \pm \sqrt{9 + 40}}{2}$$

$$x = \frac{-3 \pm \sqrt{49}}{2}$$

$$x = \frac{-3 \pm 7}{2}$$

$x = \frac{-3+7}{2}$ $\quad$ $x = \frac{-3-7}{2}$

$x = \frac{4}{2}$ $\quad$ $x = \frac{-10}{2}$

$x = 2$ $\quad$ $x = -5$

$$-2x^2 - 3x = -6$$
$$+6 \quad +6$$

a = -2
b = 3
c = 6

$$-2x^2 + 3x + 6 = 0$$

$$x = \frac{-3 \pm \sqrt{(3)^2 - 4(-2)(6)}}{2(-2)}$$

$$x = \frac{-3 \pm \sqrt{9 + 48}}{-4}$$

$$\boxed{x = \frac{-3 \pm \sqrt{57}}{-4}}$$

$\sqrt{57}$
3 19

OR

$$x = \frac{-3 + \sqrt{57}}{-4} \qquad x = \frac{-3 - \sqrt{57}}{-4}$$

$$x^2 + 9 = 0$$

a = 1
b = 0
c = 9

$$x = \frac{-0 \pm \sqrt{(0)^2 - 4(1)(9)}}{2(1)}$$

$$x = \frac{\pm \sqrt{-36}}{2}$$

$\sqrt{-36}$
-1 36
6 6
$6i$

$$x = \frac{\pm 6i}{2}$$

$$\boxed{x = \pm 3i}$$