NOTES 4-1

ALGEBRA II UNIT 4: Three Variable Systems

Jow to Write Systems of Equations (DO NOT SOLVE)

TYPE 1: Algebraic Phrase

1. The length of a rectangle is 7 more than twice the width. The perimeter is 78. Find the dimensions for this rectangle.

$\ell = 2w + 7$ (length compared to width)

$2\ell + 2w = 78$ (perimeter = 2l + 2w)

2. Juan bought 30 drinks, all either cola or root beer. The cola was equal to twice the number of cans of root beer. Write a system of equations that will determine the number of cans of cola and the number of cans of root beer that Juan bought?

$C + r = 30$

$C = 2r$

TYPE 2: Nickel and Dimes

3. Ms. Garcia has 52 coins that are all either dimes or nickels. These coins are worth $5.00. How many of each coin does she have?

$d + n = 52$ (Total # of coins)

$.10d + .05n = 5.00$ (amount the coins are worth)

.. A movie charges $5 for an adult ticket and $2 for a child ticket. The theater sold 785 tickets for $3280. How many adult tickets and how many child tickets were sold?

$5A + 2C = 3280$

$A + C = 785$

5. James has a test worth 100 points containing 30 questions. Type A questions are worth 2-points and type B questions are worth 4-points. How many of each type are on the test?

$2A + 4B = 100$

$A + B = 30$

TYPE 3: TACO AND BURRITO

6. Matt bought 3 bunnies and 5 chickens for $65. At the same farm Chuck bought 5 bunnies and 2 chickens and spent $64 . Write the system of equations to determine how much each bunnies costs and how much each chicken costs.

$3b + 5c = 65$ (Money spent by Matt)

$5b + 2c = 64$ (Money spent by Chuck)

7. Sarah bought 7 student tickets to the football game and the 2 adult tickets for $35. Valerie bought 8 student 'ickets and 1 adult tickets for $31. Write the system of equations that will determine the cost of adult tickets .nd the student tickets that they bought.

$7s + 2a = 35$

$8s + 1a = 31$

Substitution

When to use this method: When you have either x = or y = for one of your equations

Systems of equations can have one of the following:

One Solution No Solution Infinitely Many Solutions

Example 1:

$y = 2x + 3$
$3x - y = -2$

Step 1 – Identify the equation with an isolated variable.

Step 2 – Substitute the expression in Step 1 into the other equation and solve.

Step 3 – Find the other coordinate by substituting the value from step 2 into either of the original equations.

Answer – The ordered pair, (x,y), using the values in step 2 and 3.

$3x - (2x+3) = -2$
$3x - 2x - 3 = -2$
$x - 3 = -2$
$+3 \quad +3$
$x = 1$

$y = 2x + 3$
$y = 2(1) + 3$
$y = 2 + 3$
$y = 5$

$(1, 5)$

Example 2: $x = y + 5$
$y + 3x = 19$

$y + 3(y+5) = 19$
$y + 3y + 15 = 19$
$4y + 15 = 19$
$-15 \quad -15$
$\frac{4y}{4} = \frac{4}{4}$
$y = 1$

$x = y + 5$
$x = 1 + 5$
$x = 6$

$(6, 1)$

Example 3: $y = 4x + 3$
$y = x$

$4x + 3 = x$
$-4x \quad -4x$
$\frac{3}{-3} = \frac{-3x}{-3}$
$-1 = x$

$y = x$
$y = -1$

$(-1, -1)$

Example 4: $-3x - y = 5$
$x = 3y - 5$

$-3(3y-5) - y = 5$
$-9y + 15 - y = 5$
$-10y + 15 = 5$
$-15 \quad -15$
$\frac{-10y}{-10} = \frac{-10}{-10}$
$y = 1$

$x = 3y - 5$
$x = 3(1) - 5$
$x = 3 - 5$
$x = -2$

$(-2, 1)$

Example 5: $-4x + y = 6$
$y = -5x - 21$

$-4x + -5x - 21 = 6$
$-9x - 21 = 6$
$+21 \quad +21$
$\frac{-9x}{-9} = \frac{27}{-9}$
$x = -3$

$y = -5x - 21$
$y = -5(-3) - 21$
$y = 15 - 21$
$y = -6$

$(-3, -6)$

lgebra II Unit 4 Three Variable Systems

Solving 3x3 Systems by Substitution

Steps for solving systems using SUBSTITUTION:

- Step 1: Isolate one of the variables.
- Step 2: Substitute the expression from Step 1 into one of the other equations.
  - The resulting equation should have only two variables, not three.
- Step 3: Isolate a different variable. Substitute for this variable in the equation from Step 2.
- Step 4: Solve for one of the variables.
- Step 5: Plug your solution into a different equation to solve for a second variable.
- Step 6: Plug the two solutions into a different equation to solve for the third variable.
- Step 6: Write the solution as an ordered pair. (*x*, *y*, *z*)

3x3 Systems

$$\begin{cases} y = x + 4z - 5 & (1) \\ 4x + 3y - 2z = 5 & (2) \\ z = -2x + 2 & (3) \end{cases}$$

$y = x + 4(-2x+2) - 5$

$y = x - 8x + 8 - 5$

$y = -7x + 3$

Solution:

x = 0 **y =** 3 **z=** 2

(0 , 3 , 2 **)**

$4x + 3(-7x+3) - 2(-2x+2) = 5$

$4x - 21x + 9 + 4x - 4 = 5$

$-13x + 5 = 5$

$-5 \quad -5$

$-13x = 0$

$x = 0$

$y = -7x + 3$

$y = -7(0) + 3$

$y = 0 + 3$

$y = 3$

$z = -2x + 2$

$z = -2(0) + 2$

$z = 0 + 2$

$z = 2$

Check:

$3 = 0 + 4(2) - 5$

$3 = 0 + 8 - 5$

$3 = 3$ ✓

$4(0) + 3(3) - 2(2) = 5$

$0 + 9 - 4 = 5$

$5 = 5$ ✓

$2 = -2(0) + 2$

$2 = 0 + 2$

$2 = 2$ ✓

Systems of Equations Word Problems

1. The length of a rectangle is 7 more than twice the width. The perimeter is 74. Find .ne dimensions for this rectangle.

Define the variables:

L = length *W* = width

Write a system of equations:

Sentence #1 $l = 2w + 7$

Sentence #2 $2l + 2w = 74$ *(P=2L + 2W)*

Solve using substitution:

$2(2w+7) + 2w = 74$
$4w + 14 + 2w = 74$
$6w + 14 = 74$
$-14 \quad -14$
$6w = 60$
$w = 10$

$l = 2w + 7$
$l = 2(10) + 7$
$l = 20 + 7$
$l = 27$

length = 27
width = 10

. An ice cream stand sells chocolate, strawberry, and vanilla ice cream sundaes. Yesterday they sold a total of 232 ice cream sundaes. The number of vanilla they sold were 4 fewer than 3 times as many as strawberry. The number of strawberry and vanilla combined equals the number of chocolates sold. How many of each did they sell?

$C + S + V = 232$
$V = 3s - 4$
$S + V = C$

$S + 3s - 4 = c$
$4s - 4 = c$

$4(30) - 4 = c$
$120 - 4 = c$
$116 = c$

$4s - 4 + s + 3s - 4 = 232$
$8s - 8 = 232$
$+8 \quad +8$
$\frac{8s}{8} = \frac{240}{8}$
$s = 30$

$V = 3s - 4$
$V = 3(30) - 4$
$V = 90 - 4$
$V = 86$

\# of vanilla = 86 # of chocolate = 116 # of strawberry = 30

Solve using Gaussian Elimination:

*Swap R_1 & R_2

$$\left[\begin{array}{ccc|c} 2 & -1 & 2 & 10 \\ 1 & -2 & 1 & 8 \\ 3 & -1 & 2 & 11 \end{array}\right]$$

$\boxed{{}^*R_2 - 2R_1 \longrightarrow R_2}$

$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & 8 \\ 2 & -1 & 2 & 10 \\ 3 & -1 & 2 & 11 \end{array}\right]$$

$2-2(1)=0$
$-1-2(-2)=3$
$2-2(1)=0$
$10-2(8)=-6$

$\boxed{{}^*R_3 - 3R_1 \longrightarrow R_3}$

$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & 8 \\ 0 & 3 & 0 & -6 \\ 3 & -1 & 2 & 11 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & 8 \\ 0 & 3 & 0 & -6 \\ 0 & 5 & -1 & -13 \end{array}\right]$$

$3-3(1)=0$ $2-3(1)=-1$
$-1-3(-2)=5$ $11-3(8)=-13$

$x =$ 1 $y =$ −2 $z =$ 3

Solve R_2 for *y

$\frac{3y}{3}=\frac{-6}{3}$
$y=-2$

Plug *y* into R_3, solve for *z

$5y-z=-13$
$5(-2)-z=-13$
$-10-z=-13$
$+10 \quad +10$
$\frac{-z}{-1}=\frac{-3}{-1}$
$z=3$

Plug *y* & *z* into R_1, solve for *x

$x-2y+z=8$
$x-2(-2)+3=8$
$x+4+3=8$
$x+7=8$
$-7 \quad -7$
$x=1$

Set up an augmented matrix & solve using Gaussian Elimination:

$$\begin{cases} x+y+z=3 \\ x+2y+3z=0 \\ x+3y+4z=-2 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 1 & 2 & 3 & 0 \\ 1 & 3 & 4 & -2 \end{array}\right]$$

$1-1=0$
$2-1=1$
$3-1=2$
$0-3=-3$

$R_2 - R_1 \to R_2$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & 1 & 2 & -3 \\ 1 & 3 & 4 & -2 \end{array}\right]$$

$1-1=0$
$3-1=2$
$4-1=3$
$-2-3=-5$

$R_3 - R_1 \to R_3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & 1 & 2 & -3 \\ 0 & 2 & 3 & -5 \end{array}\right]$$

$0-2(0)=0$
$2-2(1)=0$
$3-2(2)=-1$
$-5-2(-3)=1$

$R_3 - 2R_2 \to R_3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & 1 & 2 & -3 \\ 0 & 0 & -1 & 1 \end{array}\right]$$

$\frac{-z}{-1}=\frac{1}{-1}$
$z=-1$

$y+2z=-3$
$y+2(-1)=-3$
$y-2=-3$
$+2 \quad +2$
$y=-1$

$x+y+z=3$
$x-1-1=3$
$x-2=3$
$+2 \quad +2$
$x=5$

$(5,-1,-1)$

$$\begin{cases} -x-5y-5z=2 \\ 4x-5y+4z=19 \\ x+5y-z=-20 \end{cases}$$

$$\left[\begin{array}{ccc|c} -1 & -5 & -5 & 2 \\ 4 & -5 & 4 & 19 \\ 1 & 5 & -1 & -20 \end{array}\right]$$

$-1(-1)=1$
$-1(-5)=5$
$-1(-5)=5$
$-1(2)=-2$

$-1(R_1) \to R_1$

$$\left[\begin{array}{ccc|c} 1 & 5 & 5 & -2 \\ 4 & -5 & 4 & 19 \\ 1 & 5 & -1 & -20 \end{array}\right]$$

$1-1=0$
$5-5=0$
$-1-5=-6$
$-20-(-2)=-18$

$R_3 - R_1 \to R_3$

$$\left[\begin{array}{ccc|c} 1 & 5 & 5 & -2 \\ 4 & -5 & 4 & 19 \\ 0 & 0 & -6 & -18 \end{array}\right]$$

$-1(4)=-4$
$-1(-5)=5$
$-1(4)=-4$
$-1(19)=-19$

$-1(R_2) \to R_2$

$$\left[\begin{array}{ccc|c} 1 & 5 & 5 & -2 \\ -4 & 5 & -4 & -19 \\ 0 & 0 & -6 & -18 \end{array}\right]$$

$-4-1=-5$
$5-5=0$
$-4-5=-9$
$-19-(-2)=-17$

$R_2 - R_1 \to R_2$

$$\left[\begin{array}{ccc|c} 1 & 5 & 5 & -2 \\ -5 & 0 & -9 & -17 \\ 0 & 0 & -6 & -18 \end{array}\right]$$

$\frac{-6z}{-6}=\frac{-18}{-6}$
$z=3$

$-5x-9z=-17$
$-5x-9(3)=-17$
$-5x-27=-17$
$+27 \quad +27$
$\frac{-5x}{-5}=\frac{10}{-5}$ $x=-2$

$x+5y+5z=-2$
$-2+5y+5(3)=-2$
$-2+5y+15=-2$
$13+5y=-2$
$-13 \quad -13$

$\frac{5y}{5}=\frac{-15}{5}$
$y=-3$

$(-2,-3,3)$

Using Matrices to Solve Word Problems

1. Juan's taco Hut sells tacos, burritos and enchiladas. Yesterday they sold a total of 954 food items. The umber of tacos they sold was 12 less than 2 times the number of burritos. The number of enchiladas they sold was exactly half as many tacos were sold. What is the amount of each food item sold.

- Define the variables: t = tacos b = burritos e = enchiladas
- Write a system of equations:

Sentence #1: $t + b + e = 954$

Sentence #2: $t = 2b - 12$

$t - 2b = -12$

Sentence #3: $e = \frac{1}{2}t$

$-\frac{1}{2}t + e = 0$

$$A = \begin{array}{c} \begin{array}{ccc} t & b & e \end{array} \\ \begin{bmatrix} 1 & 1 & 1 \\ 1 & -2 & 0 \\ -.5 & 0 & 1 \end{bmatrix} \end{array} \quad B = \begin{bmatrix} 954 \\ -12 \\ 0 \end{bmatrix}$$

- Set up the matrices & solve in the calculator:

$$\begin{bmatrix} 474 \\ 243 \\ 237 \end{bmatrix} \begin{array}{c} t \\ b \\ e \end{array}$$

474 tacos
243 burritos
237 enchiladas

John inherited $25,000 and invested part of it in a money market account, part in government bonds, and part in a retirement fund. After one year, he received a total of $1,620 in simple interest from the three investments. The money market paid 6% annually, the bonds paid 7% annually, and the retirement fund paid 8% annually. There was $6,000 more invested in the bonds than the retirement fund. Find the amount John invested in each category.

- Define the variables: m: money market
b: bonds
r: retirement
- Write a system of equations:

$m + b + r = 25{,}000$

$.06m + .07b + .08r = 1620$

$b = r + 6000$

$\hookrightarrow b - r = 6000$

$$A = \begin{array}{c} \begin{array}{ccc} m & b & r \end{array} \\ \begin{bmatrix} 1 & 1 & 1 \\ .06 & .07 & .08 \\ 0 & 1 & -1 \end{bmatrix} \end{array} \quad B = \begin{bmatrix} 25{,}000 \\ 1620 \\ 6000 \end{bmatrix}$$

- Set up the matrices & solve in the calculator

$$\begin{bmatrix} 15{,}000 \\ 8{,}000 \\ 2{,}000 \end{bmatrix}$$

$m = 15{,}000$
$b = 8{,}000$
$r = 2{,}000$