NOTES 17 - 1

Algebra II UNIT 17 Exponential and Logarithm Functions

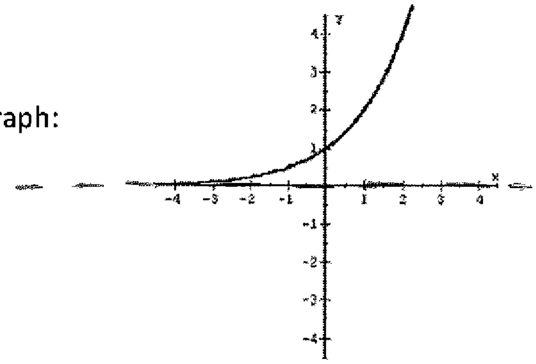
Exponential Graphs: Domain, Range, Parameter Changes, Asymptotes

Parent Function:

Name: Exponential

Equation: $y = 2^x$ $y = 10^x$ $y = e^x$

Graph:



Domain: $(-\infty, \infty)$

Range: $(0, \infty)$

Asymptote: $y = 0$

Transformations: $y = a(n)^{[b(x-h)]} + k$

n = base number (only changes the parent values, which are given)

a = V. Reflection (-); V. Stretch or Compression

b = H. Reflection (-); H. Stretch or Compression

h = Shift left or Right

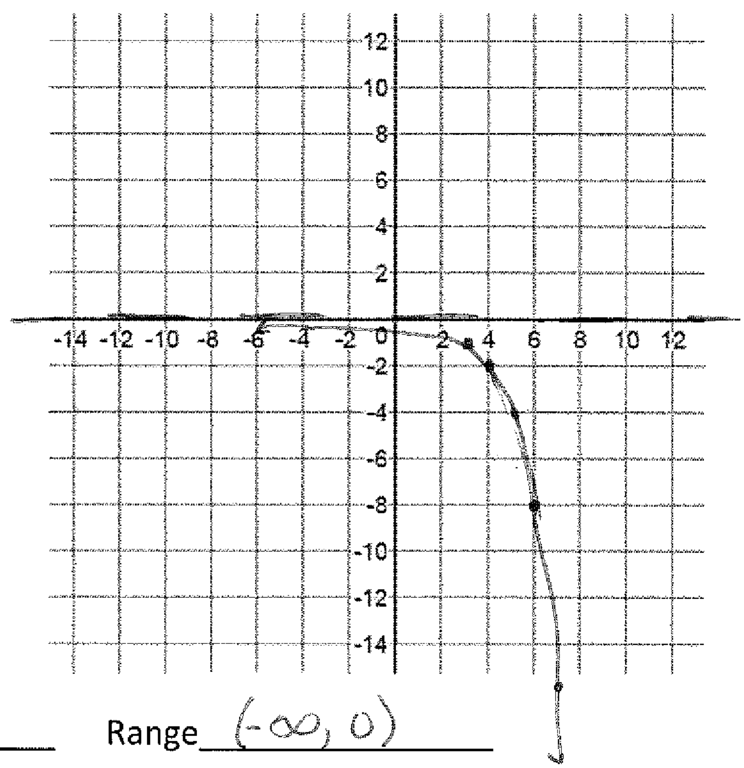
k = Shift Up or Down

Describe the transformations & graph the transformed function:

1. $y = -1(2)^{x-3}$ Base is 2

   1. V. Reflection

   2. Right 3

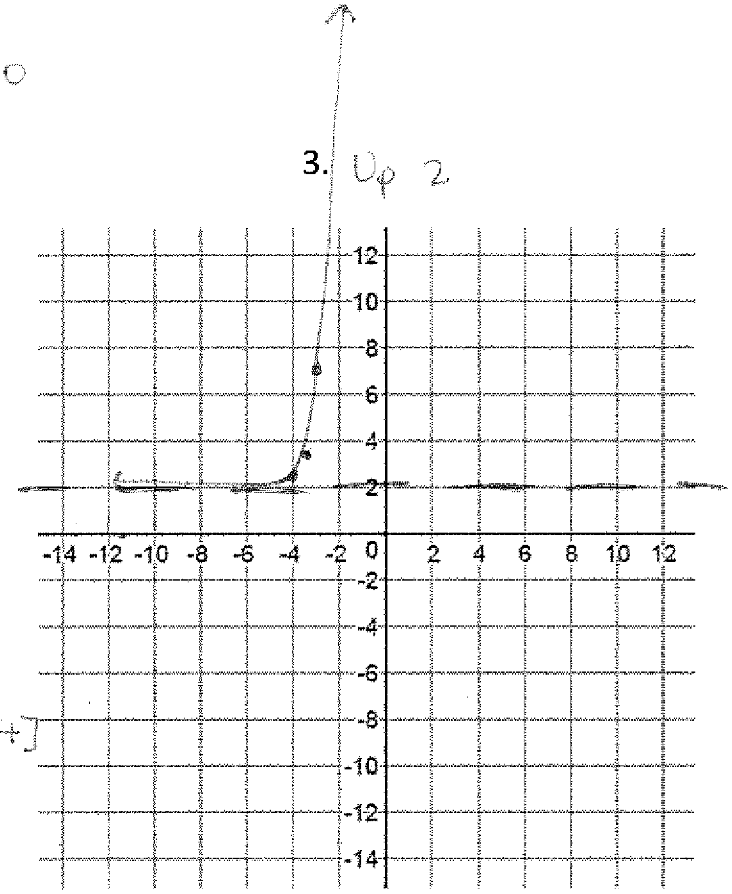


h	1/b	x parent function	y parent function	a	k
+3	* 1			* -1	+ 0
3	0	0	1	-1	-1
4	1	1	2	-2	-2
5	2	2	4	-4	-4
6	3	3	8	-8	-8
7	4	4	16	-16	-16

Asymptote $y = 0$ Domain $\mathbb{R}$ Range $(-\infty, 0)$

2. $y = (.5)(10)^{x+4} + 2$ Base is 10

1. V. Comp of 1/2 (.5) 2. Left 4 3. Up 2



h -4	1/b * 1	x parent function	y parent function	a *.5	k +2
-4	0	0	1	.5	2.5
-3.5	.5	.5	3	1.5	3.5
-3	1	1	10	5	7
-2	2	2	100	50	52

Asymptote $y = 2$ [comes from V. shift]

Domain $\mathbb{R}$

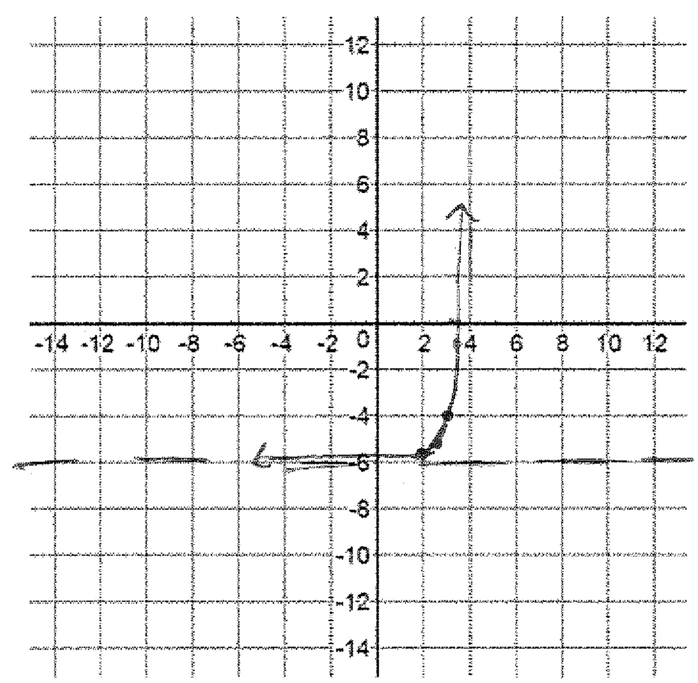
Range $(-2, \infty)$

3. $y = .25(e)^{(2x-4)} - 6$ e is the base

(*Factor first)

$.25(e)^{2(x-2)} - 6$

1. V. Comp 1/4 (.25) 2. H. Comp 1/2
3. Right 2 4. Down 6



h +2	1/b * 1/2	x parent function	y parent function	a *.25	k -6
2	0	0	1	.25	-5.75
2.5	.5	1	2	.5	-5.5
3	1	2	8	2	-4
3.5	1.5	3	20	5	-1

Asymptote $y = -6$ Domain $\mathbb{R}$ Range $(-6, \infty)$

LOGS Graphs:

Parent Function: $\log_b x = y$

Name: Logarithm

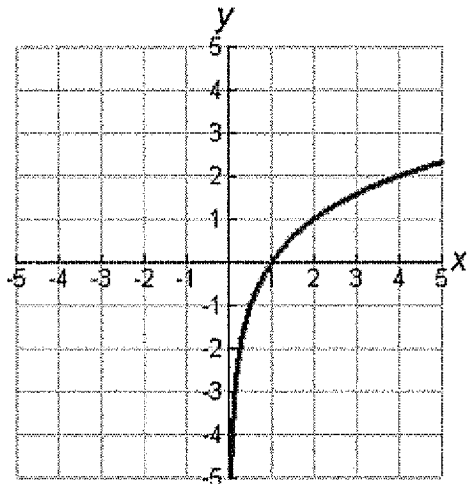
Equation:

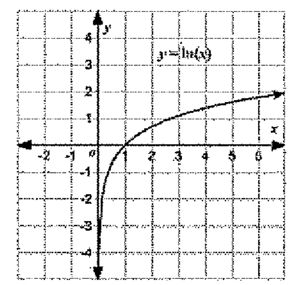
$y = \log_2 x$

$y = \log x$ → Base 10

$y = \ln x$ → Natural Log

Graph:





Domain: $(0, \infty)$

Range: $\mathbb{R}$

Asymptote: $x = 0$

$$f(x) = a\log_2[b(x-h)] + k$$

$$f(x) = a\ln\,[b(x-h)] + k$$

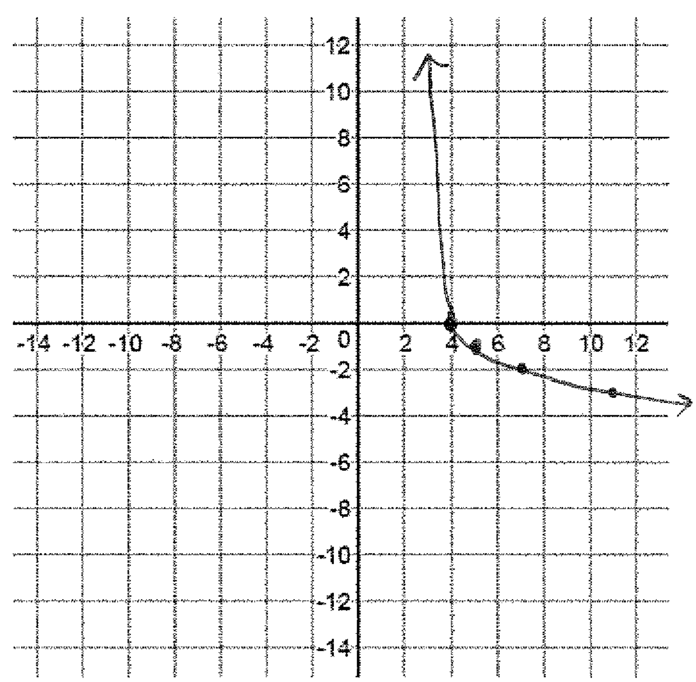
Describe the transformations & graph the transformed function:

1. $y = -log_2(x-3)$

1. V. Reflection

2. Right 3

h +3	1/b * 1	x parent function	y parent function	a * -1	k +0
4	1	1	0	0	0
5	2	2	1	-1	-1
7	4	4	2	-2	-2
11	8	8	3	-3	-3



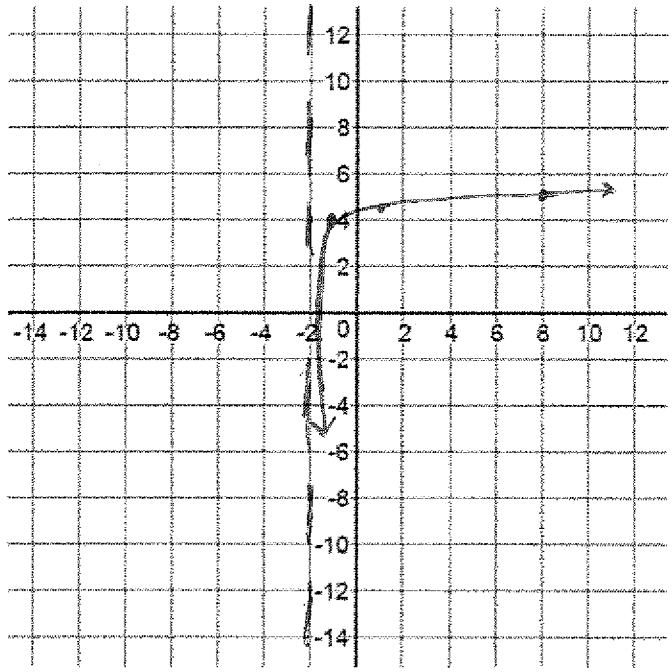
Asymptote $x = 3$ Domain $(3, \infty)$ Range $\mathbb{R}$

[comes from H. Shift]

2. $y = \frac{1}{2}log_{10}(x+2)+4$

1. V. Comp 1/2
2. Left 2
3. Up 4

h -2	1/b $*1$	x parent function	y parent function	a $*\frac{1}{2}$	k $+4$
-1	1	1	0	0	4
1	3	3	1	.5	4.5
8	10	10	2	1	5
98	100	100	3	1.5	5.5



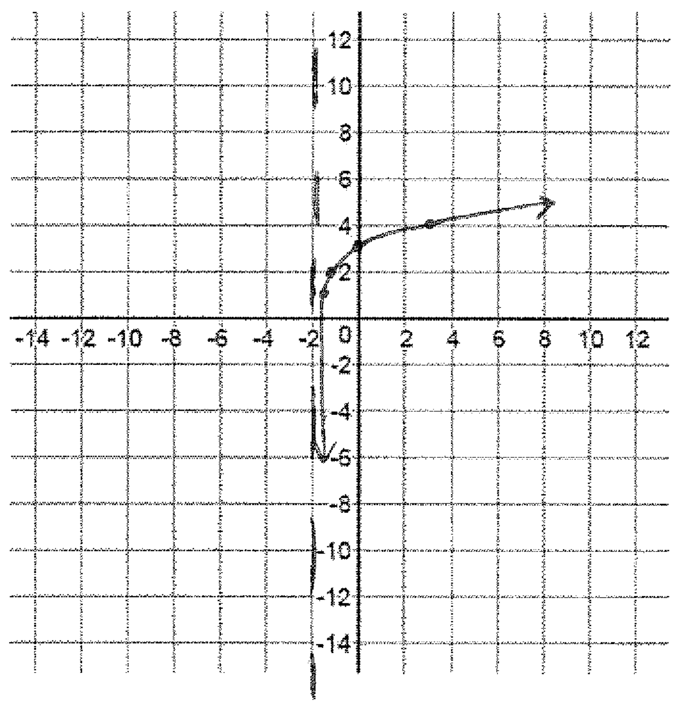
Asymptote $X=-2$ Domain $(-2,\infty)$ Range $\mathbb{R}$

3. $y = \ln(4x+8)+1$
(*Factor first)

$\ln[4(x+2)]+1$

1. H. Comp 1/4
2. Left 2
3. Up 1

h -2	1/b $*1/4$	x parent function	y parent function	a $*1$	k $+1$
-1.75	.25	1	0	0	1
-1.25	.75	3	1	1	2
0	2	8	2	2	3
3	5	20	3	3	4



Asymptote $X=-2$ Domain $(-2,\infty)$ Range $\mathbb{R}$

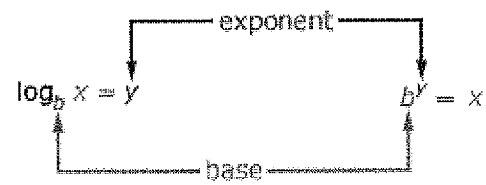
NOTES 17-2

Algebra II UNIT 17 Exponential and Logarithm Functions

Inverse operations

Addition ⟷ Subtraction
Multiplication ⟷ Division
Square ⟷ Square Root
Cubic ⟷ Cube Root
Log ⟷ Exponential

Logarithmic Form **Exponential Form**



$\log_2 16 = 4$: log base 2 of 16 equals 4.

Change Forms: Write each equation in ***exponential*** form.

$2^4 = 16$ 1. $\log_2 16 = 4$

$3^{-3} = \frac{1}{27}$ 2. $\log_3 \frac{1}{27} = -3$

$3^2 = 9$ 3. $\log_3 9 = 2$

$e^0 = 1$ 4. $\ln 1 = 0$ ln is $\log_e$

$2^3 = 8$ 5. $\log_2 8 = 3$

$e^1 = e$ 6. $\ln e = 1$

Write each equation in ***logarithmic*** form.

$\log_2 32 = 5$ 7. $2^5 = 32$ base

$\log_{10} .01 = -2$ 8. $10^{-2} = 0.01$

$\log_3 \frac{1}{9} = -2$ 9. $3^{-2} = 1/9$

$\log 100 = 2$ 10. $10^2 = 100$

$\log_5 125 = 3$ 11. $5^3 = 125$

$\log_7 1 = 0$ 12. $7^0 = 1$

Mental Math: *Evaluate* the expression. (What is the exponent?)

13. $\log_2 4$ 2
$2^? = 4$

14. $\log_5 125$ 3
$5^? = 125$

15. $\log 1000$ 3
$10^? = 1000$

16. $\log_{13} \sqrt{13}$ 1/2
$13^? = \sqrt{13}$ $\sqrt{13} = 13^{1/2}$

17. $\log_2 8$ 3
$2^? = 8$

18. $\log_5 1/5$ −1
$5^? = \frac{1}{5}$ $5^1 = 5$ $5^{-1} = \frac{1}{5}$

19. $\log_7 1$ 0
$7^? = 1$

20. $\ln 1/e$ −1
$e^? = \frac{1}{e}$

21. $\log_8 2$ 1/3
$8^? = 2$
$\sqrt[3]{8} = 2$

22. $\log_3 27$ 3
$3^? = 27$

23. $\log_{832} 1$ 0
$832^? = 1$

24. $\ln \sqrt{e}$ 1/2
$e^? = \sqrt{e}$

NOTES 17-3

Algebra II UNIT 17 Exponential and Logarithm Functions

FINDING THE INVERSE FUNCTION: Switch x and y. Solve for y.

$$\log_b a = x \leftrightarrow b^x = a$$

1. $f(x) = \log_2(x+5) - 9$

$x = \log_2(y+5) - 9$

$+9 \qquad +9$

$x + 9 = \log_2(y+5)$

$2^{(x+9)} = y + 5$

$-5 \qquad -5$

$2^{(x+9)} - 5 = y$

$f^{-1}(x) = 2^{(x+9)} - 5$

2. $g(x) = 5 \log_4 x$

$\frac{1}{5} \cdot x = 5 \log_4 y \cdot \frac{1}{5}$

$\frac{1}{5}x = \log_4 y$

$4^{1/5 x} = y$

$g^{-1}(x) = 4^{1/5 x}$

3. $h(x) = 4^{x/6}$

$x = 4^{y/6}$

$6 \cdot \log_4 x = \frac{y}{6} \cdot 6$

$6 \log_4 x = y$

$h^{-1}(x) = 6 \log_4 x$

4. $g(x) = 3^{x+2} - 7$

$x = 3^{y+2} - 7$

$+7 \qquad +7$

$x + 7 = 3^{y+2}$

$\log_3(x+7) = y + 2$

$-2 \qquad -2$

$\log_3(x+7) - 2 = y$

$g^{-1}(x) = \log_3(x+7) - 2$

5. $f(x) = \log_{1/5}(x) - 4$

$x = \log_{\frac{1}{5}}(y) - 4$

$+4 \qquad\qquad +4$

$x + 4 = \log_{\frac{1}{5}}(y)$

$\frac{1}{5}^{(x+4)} = y$

$f^{-1}(x) = \frac{1}{5}^{(x+4)}$

6. $g(x) = \log_6 \overset{[4(x+1)]}{(4x+4)}$

$x = \log_6[4(y+1)]$

$\frac{1}{4} \cdot 6^x = 4(y+1) \cdot \frac{1}{4}$

$\frac{1}{4} \cdot 6^x = y + 1$

$-1 \qquad -1$

$\frac{1}{4} \cdot 6^x - 1 = y$

$g^{-1}(x) = \frac{1}{4} \cdot 6^x - 1$

7. $h(x) = (3^x + 4)^{1/3}$

$x^3 = \left[(3^y + 4)^{1/3}\right]^3$

$x^3 = 3^y + 4$

$-4 \qquad -4$

$x^3 - 4 = 3^y$

$\log_3(x^3 - 4) = y$

$h^{-1}(x) = \log_3(x^3 - 4)$

8. $p(x) = (2^x + 3)^{1/4} - 5$

$x = (2^y + 3)^{1/4} - 5$

$+5 \qquad\qquad +5$

$(x+5)^4 = \left[(2^y + 3)^{1/4}\right]^4$

$(x+5)^4 = 2^y + 3$

$-3 \qquad -3$

$(x+5)^4 - 3 = 2^y$

$\log_2[(x+5)^4 - 3] = y$

$p^{-1}(x) = \log_2[(x+5)^4 - 3]$

NOTES 17-4

Algebra II UNIT 17 Exponential and Logarithm Functions

Remember:
The inverse of a root is an exponent
To cancel out a fraction we use the reciprocal

Part 1: Solving log equations

1. Isolate the log function
2. Cancel log with exponential
3. Solve for x

Examples *(#1-3: the log part is already isolated)*

1. $\log_x 5 = \frac{1}{2}$ OYO : $\log_x 9 = 2$

$(x^{1/2})^2 = (5)^2$

$x = 25$

2. $\log_{10} x = 3$ OYO : $\log_{11} x = 2$

$10^3 = x$

$1000 = x$

3. $\log_4 \frac{1}{16} = x$ OYO : $\log_{\frac{1}{4}} 64 = x$

$4^x = \frac{1}{16}$ $4^2 = 16$

$x = -2$

(#4-6: Isolate the log before canceling it out!)

4. $\frac{4 \log_x 81}{4} = \frac{8}{4}$ OYO : $2 \log_x 8 = 6$

$\log_x 81 = 2$

$\sqrt{x^2} = \sqrt{81}$

$x = 9$

5. $-3 + \log_x 8 = 0$ OYO : $-5 + \log_x 1 = 2$

$+3 \quad\quad +3$

$\log_x 8 = 3$

$\sqrt[3]{x^3} = \sqrt[3]{8}$

$x = 2$

Part 2: When mental math isn't enough....

Change of Base Formula: $\log_b x = \dfrac{\log x}{\log b}$

Evaluate. Use the ***change of base formula*** to find the value of each log expression with you calculator.

6. $\log_2 7$ $\underline{2.807}$

$\dfrac{\log 7}{\log 2}$

7. $\log_3 11$ $\underline{2.183}$

$\dfrac{\log 11}{\log 3}$

8. $\log_{33} 5$ $\underline{.460}$

$\dfrac{\log 5}{\log 33}$

Part 3: Solving exponential equations

1. Get the Exponential part alone
2. Cancel exponential with log
3. Use change of base formula
4. Round to the nearest thousandths place.

9. $4^{x-8} = 87$

$\log_4 87 = x - 8$

$\dfrac{\log 87}{\log 4} = x - 8$
$+8 \qquad +8$

$\dfrac{\log 87}{\log 4} + 8 = x$

$x = 11.221$

OYO: $3^{2x-1} = 12$

10. $5^{-x} = 50$

$\log_5 50 = -x$

$-1 \cdot \dfrac{\log 50}{\log 5} = \dfrac{-x}{-1}$

$x = -2.431$

OYO: $2^{x+1} = 7$

11. $5(1.5)^{x} - 5 = 335$

$\quad\quad +5 \quad +5$

$$\frac{5(1.5)^x}{5} = \frac{340}{5}$$

$$(1.5)^x = 68$$

$$\log_{1.5} 68 = x$$

$$\frac{\log 68}{\log 1.5} = x \qquad x = 10.407$$

OYO: $16^{x-7} + 5 = 24$

12. $2(3)^{-2k+8} - 3 = 63$

$\quad\quad +3 \quad +3$

$$\frac{2(3)^{-2k+8}}{2} = \frac{66}{2}$$

$$(3)^{-2k+8} = 33$$

$$\log_3 33 = -2k+8$$

$$\frac{\log 33}{\log 3} = -2k+8$$

$$-8 \qquad\qquad -8$$

$$\frac{-4.817}{-2} = \frac{-2k}{-2}$$

$$K = 2.409$$

OYO: $2(3)^{-4x-1} - 4 = 10$

13. $4e^{8x+8} - 3 = 23$

$\quad\quad +3 \quad +3$

$$\frac{4e^{8x+8}}{4} = \frac{26}{4}$$

$$e^{8x+8} = 6$$

$$\ln 6 = 8x + 8$$

$$-8 \qquad\quad -8$$

$$\frac{\ln 6 - 8}{8} = \frac{8x}{8}$$

$$x = -.776$$

OYO: $3e^{2x+5} + 5 = 16$

NOTES 17 - 5

Algebra II UNIT 17 Exponential and Logarithm Functions

COMPOUND INTEREST/Percent change

A accrued amount (total)

P principle (starting amount)

r rate as a decimal

n number of times interest is compounded (calulated)

t time in years

Formula:

$$A = P\left(1 + \frac{r}{n}\right)^{(n \cdot t)}$$

verbal	n =
annually	1
quarterly	4
monthly	12
Semi-annually	2
Bi-monthly	6
weekly	52
daily	365

P r = .065 n = 4 t = 3

1. $500 is invested at 6.5% annual interest, compounded **quarterly**, for 3 years.

EQUATION $A = 500\left(1 + \frac{.065}{4}\right)^{4 \cdot 3}$ answer $606.70

$500(1 + .01625)^{12}$

$500(1.01625)^{12}$

$500(1.21341) = 606.70$

r = .09

2. How much money must be deposited in an account that pays 9% annual interest, compounded **semi-annually**, to have a balance of $1000 after three years?

n = 2 A = 1000 t = 3

EQUATION $1000 = P\left(1 + \frac{.09}{2}\right)^{2 \cdot 3}$ Amount Deposited $767.90

$1000 = P(1.045)^6$

$1000 = P(1.30226)$

$P = 767.90$

3. You invested $5000 and it was **depreciating** by 8.3% per year. How much would you have left after 6 years? *(if there is no word for n then n = 1)*

P −.083 t = 6

EQUATION $A = 5000\left(1 - \frac{.083}{1}\right)^{1 \cdot 6}$ answer $2972.94

$A = 5000(.917)^6$

$A = 5000(.594588)$

$A = 2972.94$

Continuous Change $A = Pe^{rt}$

4. Mr. Daniels invested \$8000 in an account paying 7.25% interest compounded continuously. $.0725$

Equation: $A = 8000e^{.0725t}$

How long will it take the money in the account to **increase by \$2000?** total of 8,000 + 2,000 = 10,000

$\frac{10{,}000}{8000} = \frac{8000e^{.0725t}}{8000}$

$1.25 = e^{.0725t}$

$\ln 1.25 = .0725t$

$\frac{.22314}{.0725} = \frac{.0725t}{.0725}$

$t = 3.1$ years → will happen in year 4.

How long will it take the money in the account to **double**?

$\frac{16{,}000}{8000} = \frac{8000e^{.0725t}}{8000}$

$2 = e^{.0725t}$

$\ln 2 = .0725t$

$\frac{.69315}{.0725} = \frac{.0725t}{.0725}$ $t = 9.6$ Will happen in year 10.

5. A population of 450 animals decreases continuously at an annual rate of 12% per year.

Equation: $A = 450e^{-.12t}$

How long before there are less than 300 animals left?

$\frac{300}{450} = \frac{450e^{-.12t}}{450}$

$.66667 = e^{-.12t}$

$\ln .66667 = .12t$

$\frac{-.40546}{-.12} = \frac{-.12t}{-.12}$ $t = 3.4$ Will happen in year 4

How long before there are less than 100 animals left?

$\frac{100}{450} = \frac{450e^{-.12t}}{450}$

$.2222 = e^{-.12t}$

$\ln .2222 = -.12t$

$\frac{-1.5042}{-.12} = \frac{-.12t}{-.12}$ $t = 12.5$ Will happen in year 13

Growth	**Decay**
$y = C \cdot \underline{b}^x$ where $b > 1$	$y = C \cdot \underline{b}^x$ where $b < 1$
C= initial amount	C= initial amount
b= constant ratio	b= constant ratio
x = time period	x = time period

6. The number of cells of a certain ***bacteria*** **doubles** every minute. 50 bacteria cells were identified in a patient's body. Find the number of cells at each time.

2

Equation $y = 50 \cdot 2^x$ **growth** or **decay**

a. 1 minute 100
$y = 50 \cdot 2^1$

b. 5 minutes 1600
$y = 50 \cdot 2^5$

c. 14-minutes 819,200
$y = 50 \cdot 2^{14}$

d. How long before there are 51,200 bacteria cells in the body? 10 mins

$$\frac{51,200}{50} = \frac{50 \cdot 2^x}{50}$$

$1024 = 2^x$

$\log_2 1024 = x$

$\frac{\log 1024}{\log 2} = x \quad x = 10$

7. 400 mg of ***penicillin*** was given to a patient to fight the bacteria described above. When this particular type of penicillin is injected into a person's bloodstream, the amount left after 1 hour is about **half** of the original amount. Find the amount left in the person's bloodstream at each of the given times.

Equation $y = 400 \cdot \frac{1}{2}^x$ **growth** or **decay**

a. 3 hours 50 mg
$y = 400 \cdot \frac{1}{2}^3$

b. 6 hrs 6.25 mg
$y = 400 \cdot \frac{1}{2}^6$

c. 16 hours .006 mg
$y = 400 \cdot \frac{1}{2}^{16}$

d. How long before the bloodstream has only half of the penicillin remaining? 1 hour

$$\frac{200}{400} = \frac{400 \cdot \frac{1}{2}^x}{400}$$

$.5 = \frac{1}{2}^x$

$\log_{1/2}(.5) = x \quad \frac{\log .5}{\log .5} = x \quad x = 1$