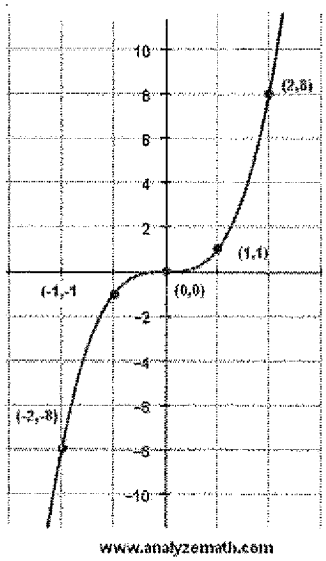
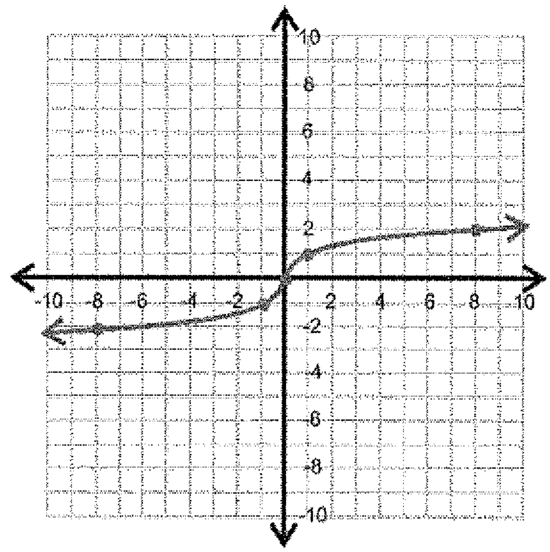
NOTES 16-1

Algebra II Unit 16 Analyze Cubics and Their Inverse





CUBIC: $y = a[b(x-h)]^3 + k$ **CUBE ROOT:** $y = a\sqrt[3]{b(x-h)} + k$

(-2 , -8)	(-8 , -2)
(-1 , -1)	(-1 , -1)
(0 , 0)	(0 , 0)
(1 , 1)	(1 , 1)
(2 , 8)	(8 , 2)

a = vertical reflection; vertical stretch or compression

b = horizontal reflection, horizontal stretch or compression (use reciprocal of b)

h = shift left or right (opposite of what it appears)

k = shift up or down

Describe the Transformations, fill in the table, and graph the function:

$f(x) = 3[\frac{1}{2}(x-1)]^3 - 2$

What type of function is f(x)? Cubic

1. V. Stretch of 3
2. H. Stretch of 2
3. Right 1
4. Down 2

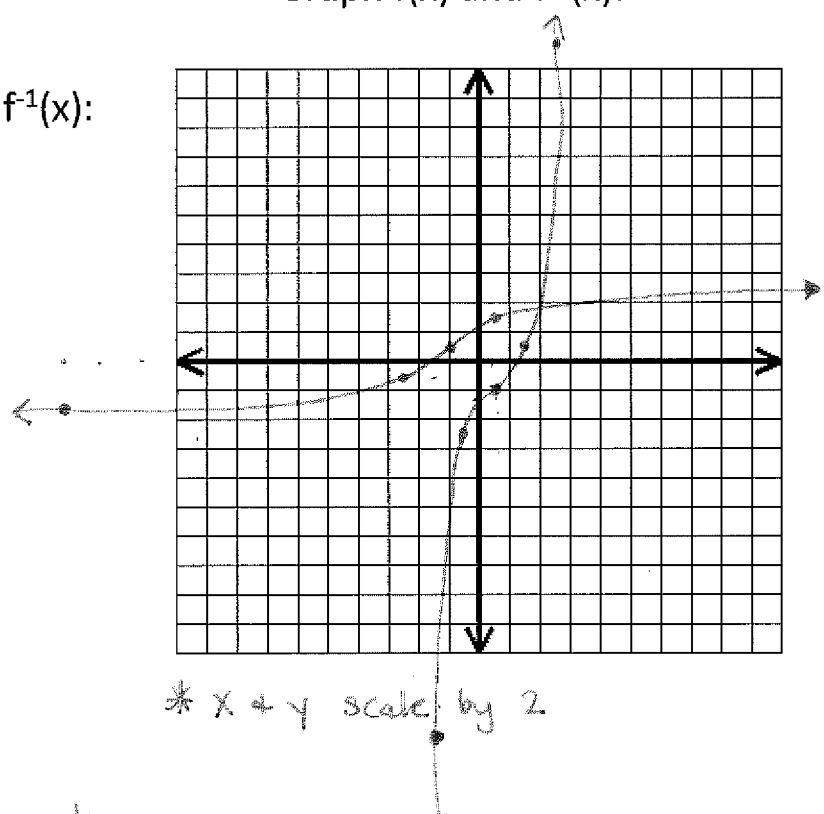
Domain: $\mathbb{R}$

Range: $\mathbb{R}$

(*Graph new function using outer table rows)

h +/- (+1)	1/b x 2	x parent function	y parent function	a x 3	k +/- (−2)
−3	−4	-2	-8	−24	−26
−1	−2	-1	-1	−3	−5
1	0	0	0	0	−2
3	2	1	1	3	1
5	4	2	8	24	22

Graph f(x) and $f^{-1}(x)$:



* x & y scale by 2

Fill in the table for the inverse function $f^{-1}(x)$:

x	y
−26	−3
−5	−1
−2	1
1	3
22	5

What type of function is $f^{-1}(x)$? cube root

Domain: $\mathbb{R}$ Range: $\mathbb{R}$

$f(x) = -\frac{1}{2}\sqrt[3]{2(x+4)} + 1$

What type of function is f(x)? Cube root

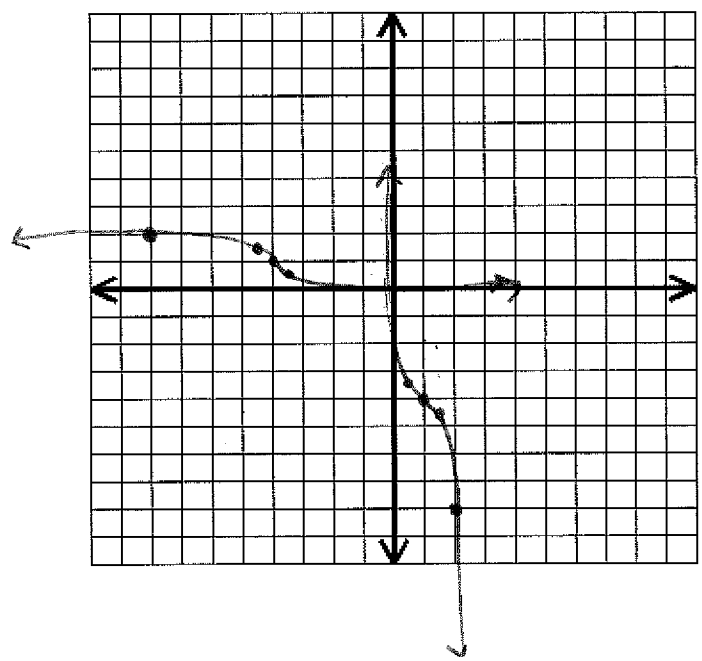
Domain: $\mathbb{R}$

Range: $\mathbb{R}$

1. V. Reflection
2. Left 4
3. V. Comp 1/2
4. Up 1
5. H. Comp 1/2

h +/- (-4)	1/b x 1/2	x parent function	y parent function	a x -1/2	k +/- (+1)
-8	-4	-8	-2	1	2
-4.5	-.5	-1	-1	.5	1.5
-4	0	0	0	0	1
-3.5	.5	1	1	-.5	.5
0	4	8	2	-1	0

Graph f(x) and $f^{-1}(x)$:



Fill in the table for the inverse function $f^{-1}(x)$:

x	y
2	-8
1.5	-4.5
1	-4
.5	-3.5
0	0

What type of function is $f^{-1}(x)$? Cubic

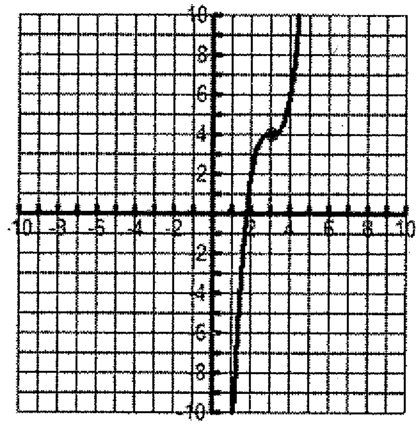
Domain: $\mathbb{R}$ Range: $\mathbb{R}$

Identifying Transformations from graphs:

Example 1:

a. Describe the transformations shown compared to the cubic parent function.

1. Right 3
2. Up 4



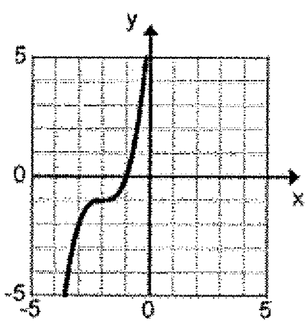
b. Write an equation for the cubic function represented by the graph.

$y = (x-3)^3 + 4$

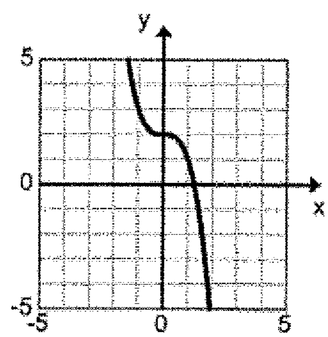
Example 2:

Which graph or graphs below show a vertical reflection of the cubic parent function? B & C

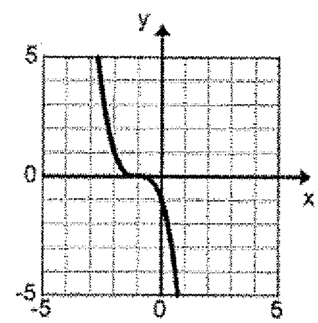
A.



B.



C.

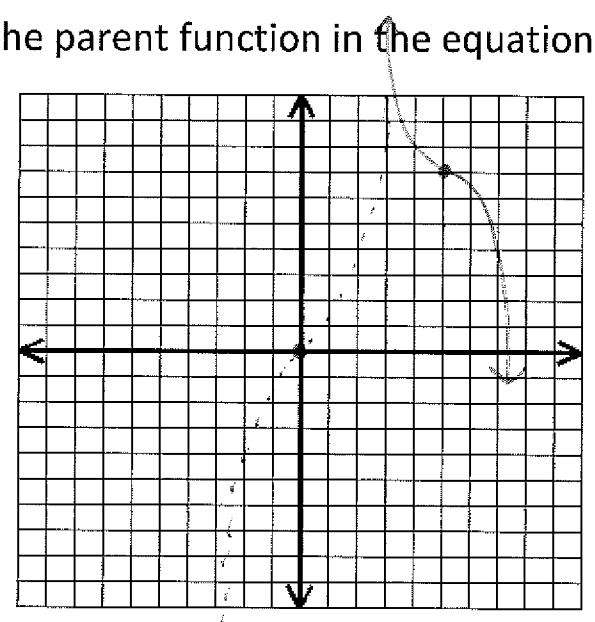


Example 3: Describe the transformations to the parent function in the equation, $y = -\sqrt[3]{x-5} + 7$. Sketch the graph.

1. V. Reflection

2. Right 5

3. Up 7



NOTES 16-2

Algebra II Unit 16 Analyze Cubics and their Inverses

Find the Inverse function **using inverse operations.** Write the inverse function using function notation: $f^{-1}(x)$

Two methods for finding inverse functions:

- *Show steps*
  Switch x and y. Use inverse operations to solve for x.
  OR
- *List steps*
  Perform the inverse of each operation in the opposite position. (inside vs outside)

$f(x) = -\frac{1}{2}\sqrt[3]{x+3} + 5$ *(Show Steps)*

$f^{-1}(x) = [-2(x-5)]^3 - 3$

$x = -\frac{1}{2}\sqrt[3]{y+3} + 5$

$-5 \qquad -5$

$-2(x-5) = -\frac{1}{2}\sqrt[3]{y+3} \quad \cdot -2$

$[-2(x-5)]^3 = \left(\sqrt[3]{y+3}\right)^3$

$[-2(x-5)]^3 = y + 3$

$-3 \qquad -3$

$[-2(x-5)]^3 - 3 = y$

$h(x) = \sqrt[3]{2x-8} - 1$ *(List Steps)*

$h^{-1}(x) = \frac{1}{2}(x+1)^3 + 4$

*Factor out the coefficient of the variable !

Factored equation: $\sqrt[3]{2(x-4)} - 1$

1. Add 1 (inside)
2. Cube
3. Multiply 1/2 (outside)
4. Add 4 (outside)

$\frac{1}{2}(x+1)^3 + 4$

OYO: $g(x) = -\frac{1}{3}(x-7)^3 + 2$

(Show or list steps)

$g^{-1}(x) = \sqrt[3]{-3(x-2)} + 7$

OYO: $h(x) = (3x-18)^3 - 7$

(*Factor out the coefficient)
(Show or list steps)

$h^{-1}(x) = \frac{1}{3}\sqrt[3]{x+7} + 6$

Verify that the functions are inverse **using Composite Functions**. SHOW your WORK and then answer **yes or no.**

- Plug one function into the other, if the composite functions simplifies to x, then the functions are inverses.

1. Verify that $f(x) = \sqrt[3]{x-5} + 2$ and $g(x) = (x-2)^3 + 5$ are inverse of each other.

f(g(x))	*g(f(x))*
g gets plugged in for x $\sqrt[3]{(x-2)^3+5-5} + 2$ $\sqrt[3]{(x-2)^{\cancel{3}}} + 2$ $x - \cancel{2} + \cancel{2}$ x $\boxed{\text{Yes}}$	f gets plugged in for x $(\sqrt[3]{x-5} + 2 - 2)^3 + 5$ $(\sqrt[3]{x-5})^{\cancel{3}} + 5$ $x - \cancel{5} + \cancel{5}$ x

2. Verify that $f(x) = x^3 - 7$ and $g(x) = \frac{\sqrt[3]{x}}{5}$ are inverses of each other.

f(g(x))	*g(f(x))*
$\left(\frac{\sqrt[3]{x}}{5}\right)^3 - 7$ $\frac{x}{125} - 7$ doesn't simplify to x $\boxed{\text{No}}$	**If the f(g(x)) does not Simplify to x, then we do not have to check g(f(x)).*