NOTES 15 - 1

ALGEBRA II UNIT 15 Simplify and Solve rational exponents and roots

Part 1: Simplifying Radicals

When simplifying a square root:

1. Find the factors of the radicand (expression under the radical).
2. For every pair of factors, bring one to the outside of the radical.
3. If there is a coefficient in front of the radical already, the number will be multiplied with the coefficient.
4. The remaining factors under the radical will get multiplied back together.

When simplifying any other root, the steps are the same, but you will look for groups of factors that match the number of the root.

1. $\sqrt{60n^3}$

2 30, 2 15, 3·5; nn n

$\boxed{2n\sqrt{15n}}$

2. $-3\sqrt{144x^6}$

2 72, 2 36, 2 18, 2 9, 3 3; xx xx xx

$-3\cdot2\cdot2\cdot3\cdot x^3$

$\boxed{-36x^3}$

3. $5\sqrt{28z^7}$

2 14, 2 7; zz zz zz z

$5\cdot2\cdot z^3\sqrt{7}$

$\boxed{10z^3\sqrt{7}}$

4. $-\sqrt{75xy^5}$

3 25, 5 5; x yy yy y

$\boxed{-5y^2\sqrt{3x}}$

5. $\sqrt{200x^4y^6}$

2 100, 2 50, 2 25, 5 5; xx xx yy yy yy

$2\cdot5\cdot x^2y^3\sqrt{2}$

$\boxed{10x^2y^3\sqrt{2}}$

6. $\sqrt[4]{128n^8}$

2 64, 2 32, 2 16, 2 8, 2 4, 2 2; nnnn nnnn

$\boxed{2n^2\sqrt[4]{8}}$

7. $\sqrt[3]{24m^3}$

2 12, 2 6, 2 3; mmm

$\boxed{2m\sqrt[3]{3}}$

8. $\sqrt[3]{56x^5y}$

2 28, 2 14, 2 7; xxx xx y

$\boxed{2x\sqrt[3]{7x^2y}}$

9. $\sqrt[5]{224r^7}$

2 112, 2 56, 2 28, 2 14, 2 7; rrrrr rr

$\boxed{2r\sqrt[5]{7r^2}}$

NOTES 15 - 2

ALGEBRA II UNIT 15 **Simplify and Solve Rational Exponents and Roots**

Examples: Solve. Identify EXTRANEOUS SOLUTIONS

- To solve equations, we use inverse operations.
- The inverse operation for a square root is a square.
- The inverse operation for a cube root is a cube.

Example 1: $\sqrt{x} = 3$

$(\sqrt{x})^2 = (3)^2$

$\boxed{x = 9}$

Example 2: $\sqrt{x} = -3$

$(\sqrt{x})^2 = (-3)^2$

$x = 9$

$\sqrt{9} \neq -3$

This equation has no solution because the answer to a square root is always a positive number.

Solving Equations with Radicals
Step 1 **Isolate the radical**. Make sure that one radical term is alone on one side of the equation.
Step 2 **Apply the power rule**. Raise both sides of the equation to a power that is the same as the index of the radical.
Step 3 **Solve** the resulting equation; if it still contains a radical, repeat Steps 1 and 2.
Step 4 **Check for extraneous solutions.** When solving equations with an ***even*** root, you must plug both solutions back in to check for solutions that do not work.

1. $\sqrt[3]{4x-8} - 4 = 0$

$\quad\quad\quad\;\; +4 \quad +4$

$(\sqrt[3]{4x-8})^3 = (4)^3$

$4x - 8 = 64$

$\quad +8 \quad +8$

$\frac{4x}{4} = \frac{72}{4}$

$\boxed{x = 18}$

OYO: $3\sqrt[3]{2x+4} - 5 = 13$

$\quad\quad\quad +5 \quad +5$

$\frac{3\sqrt[3]{2x+4}}{3} = \frac{18}{3}$

$(\sqrt[3]{2x+4})^3 = (6)^3$

$2x + 4 = 216$

$\quad -4 \quad -4$

$\frac{2x}{2} = \frac{212}{2}$

$\boxed{x = 106}$

2. $2\sqrt{2k+40} - \sqrt{-16-4k} = 0$

$+\sqrt{-16-4k} \quad +\sqrt{-16-4k}$

$(2\sqrt{2k+40})^2 = (\sqrt{-16-4k})^2$

$4(2k+40) = -16-4k$

$8k+160 = -16-4k$

$+4k \qquad +4k$

$12k+160 = -16$

$-160 \quad -160$

$12k = -176$

$\frac{12k}{12} = \frac{-176}{12}$

$\boxed{x = -14.67}$

OYO: $\sqrt{4x} - \sqrt{x+3} = 0$

$+\sqrt{x+3} \quad +\sqrt{x+3}$

$(\sqrt{4x})^2 = (\sqrt{x+3})^2$

$4x = x+3$

$-x \quad -x$

$\frac{3x}{3} = \frac{3}{3}$

$\boxed{x=1}$

3. $(\sqrt{-14x+2})^2 = (x-3)^2$

$-14x+2 = (x-3)(x-3)$

$-14x+2 = x^2-3x-3x+9$

$-14x+2 = x^2-6x+9$

$+14x \qquad +14x$

$2 = x^2+8x+9$

$-2 \qquad -2$

$0 = x^2+8x+7$

$0 = (x+7)(x+1)$

7	8
x	+
7·1	7+1

$\boxed{x=-7}$ $\boxed{x=-1 \text{ ext}}$

$\sqrt{-14(-7)+2} = -7-3$

$10 = 10$

$\sqrt{-14(-1)+2} = -1-3$

$4 = -3$

OYO: $(\sqrt{4-2x-x^2})^2 = (x+2)^2$

$4-2x-x^2 = (x+2)(x+2)$

$4-2x-x^2 = x^2+4x+4$

$-4+2x+x^2 \quad +x^2+2x-4$

$0 = 2x^2+6x$

$0 = 2x(x+3)$

$\frac{2x}{2} = \frac{0}{2} \qquad x+3=0$

$-3 \quad -3$

$\boxed{x=0}$ $\boxed{x=-3 \text{ ext}}$

$\sqrt{4-2(0)-(0)^2} = 0+2$

$\sqrt{4} = 2$

$2 = 2$

$\sqrt{-4-2(-3)-(-3)^2} = -3+2$

$\sqrt{-7} \neq -1$

4. $\sqrt{5-x} - 1 = x$

$+1 \quad +1$

$(\sqrt{5-x})^2 = (x+1)^2$

$5-x = x^2+2x+1$

$-5+x \qquad +x-5$

$0 = x^2+3x-4$

$0 = (x+4)(x-1)$

-4	3
x	+
4·-1	4+-1

$\boxed{x=-4 \text{ ext}}$ $\boxed{x=1}$

$\sqrt{5--4} \; -1 = -4$

$\sqrt{9}-1$

$3-1$

$2 \neq -4$

$\sqrt{5-1} \; -1 = 1$

$\sqrt{4}-1$

$2-1$

$1 = 1$ ✓

OYO: $\sqrt{2x+5} + 5 = x$

$-5 \quad -5$

$(\sqrt{2x+5})^2 = (x-5)^2$

$2x+5 = x^2-10x+25$

$-2x-5 \qquad -2x-5$

$0 = x^2-12x+20$

$0 = (x-10)(x-2)$

20	-12
x	+
-10·-2	-10+-2

$\boxed{x=10}$ $\boxed{x=2 \text{ ext}}$

$\sqrt{2(10)+5} + 5 = 10$

$\sqrt{25}+5$

$5+5 = 10$

$10 = 10$ ✓

$\sqrt{2(2)+5} + 5 = 2$

$\sqrt{9}+5$

$3+5$

$8 \neq 2$

NOTES 15 - 3

ALGEBRA II UNIT 15 Simplify and Solve Rational Exponents and Roots

RULE	Example 1	Example 2
$a^m \cdot a^n = a^{m+n}$ When multiplying like bases add the exponents	$x^{1/2} \cdot x^{5/2} = x^{1/2+5/2} = x^{6/2} = x^3$	$x^{1/3} \cdot x^{7/3} = x^{1/3+7/3} = x^{8/3}$
$\frac{a^m}{a^n} = a^{m-n}$ When dividing like bases subtract the exponents	$\frac{x^{5/4}}{x^{3/4}} = x^{5/4-3/4} = x^{2/4} = x^{1/2}$	$\frac{x^{3/5}}{x^{2/5}} = x^{3/5-2/5} = x^{1/5}$
$(a^m)^n = a^{m*n}$ When raising an exponent to another power multiply the exponents	$(x^{3/4})^{1/2} = x^{3/4 \cdot 1/2} = x^{3/8}$	$(x^{5/7})^{1/3} = x^{5/7 \cdot 1/3} = x^{5/21}$
$(ab)^m = a^m b^m$ When raising an expression to a power, each term gets raised to the power (similar to distributing)	$(x^{1/2} y^{1/4})^3 = x^{1/2 \cdot 3} y^{1/4 \cdot 3} = x^{3/2} y^{3/4}$	$(x^{1/3} y^{1/5})^{1/2} = x^{1/3 \cdot 1/2} y^{1/5 \cdot 1/2} = x^{1/6} y^{1/10}$
$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$ When a fraction is raised to a power, both the numerator and denominator get raised to the power	$\left(\frac{a^{2/3}}{b^{1/2}}\right)^{1/4} = \frac{a^{2/3 \cdot 1/4}}{b^{1/2 \cdot 1/4}} = \frac{a^{2/12}}{b^{1/8}} = \frac{a^{1/6}}{b^{1/8}}$	$\left(\frac{x^{3/4}}{y^{1/3}}\right)^{1/2} = \frac{x^{3/4 \cdot 1/2}}{y^{1/3 \cdot 1/2}} = \frac{x^{3/8}}{y^{1/6}}$
$a^{-n} = \frac{1}{a^n}$ To make an exponent positive, move it to the other part of the fraction	$-5x^{-2/3} = \frac{-5}{x^{2/3}}$; $\frac{1}{y^{-5/8}} = y^{5/8}$	$-12x^3y^{-1/2} = \frac{-12x^3}{y^{1/2}}$
$a^0 = 1$ Any expression (other than zero) raised to the zero power equals 1	$(abcdef)^0 = 1$	$(419{,}186x^{27})^0 = 1$

Properties of <u>Rational</u> Exponents $a^{\frac{m}{n}} = \sqrt[n]{a^m}$

$12^{\frac{1}{3}} = \left(\sqrt[3]{12}\right)^1 = \sqrt[3]{12}$

$\sqrt[4]{7^3} = 7^{3/4}$

Write in radical form:

1. $12^{\frac{1}{2}}$

$\left(\sqrt{12}\right)^1 = \sqrt{12}$

2. $5^{\frac{4}{3}}$

$\left(\sqrt[3]{5}\right)^4$

3. $(6x)^{-\frac{1}{2}}$

$\frac{1}{(6x)^{1/2}} = \frac{1}{\left(\sqrt{6x}\right)^1} = \frac{1}{\sqrt{6x}}$

Write in exponential form:

4. $\sqrt[4]{3}$

$3^{1/4}$

5. $(\sqrt[5]{2})^3$

$2^{3/5}$

6. $\frac{1}{(\sqrt{5x})^6}$

$\frac{1}{(5x)^{6/2}} = (5x)^{-6/2}$

Simplify:

7. $16^{\frac{1}{2}}$

$\left(\sqrt{16}\right)^1 = \sqrt{16} = 4$

8. $25^{\frac{3}{2}}$

$\left(\sqrt{25}\right)^3 = (5)^3 = 125$

9. $(64x^8)^{\frac{3}{2}}$

$\left(\sqrt{64x^8}\right)^3 = (8x^4)^3$

$= 512x^{12}$

10. $(100n^4)^{\frac{-1}{2}}$

$\frac{1}{(100n^4)^{1/2}}$

$\frac{1}{\sqrt{100n^4}}$

$\frac{1}{10n^2}$

NOTES 15 -4

PEMDAS

ALGEBRA II UNIT 15 Simplify and Solve Rational Exponents and Roots

Solving Equations with Rational Exponents

Step 1 **Isolate the parentheses with the rational exponent.** Make sure that one parenthesis is alone on one side of the equation.

Step 2 **Apply the power rule.** Raise both sides of the equation to the reciprocal of the power. (For example, 3/2 → 2/3)

Step 3 **Solve** the resulting equation; if it still contains a rational exponent, repeat Steps 1 and 2.

Step 4 **Check for extraneous solutions.** When solving equations with an ***even*** denominator, you must plug both solutions back in to check for solutions that do not work.

Solve each equation and check for extraneous solutions.

1. $3(x+1)^{\frac{3}{2}}+4=28$

$-4 \quad -4$

$\frac{3(x+1)^{3/2}}{3}=\frac{24}{3}$

$\left[(x+1)^{3/2}\right]^{2/3}=8^{2/3}$

$x+1=\sqrt[3]{8}^{\,2}$

$x+1=2^2$

$x+1=4$

$-1 \quad -1$

$\boxed{x=3}$

Check:

$3(3+1)^{3/2}+4=28$

$3(4)^{3/2}+4=28$

$3\cdot\sqrt{4}^{\,3}+4=28$

$3\cdot 2^3+4=28$

$3\cdot 8+4=28$

$24+4=28$

$28=28$ ✓

2. $\left[(2x-7)^{\frac{1}{3}}\right]^3=(3)^3$

$2x-7=9$

$+7 \quad +7$

$\frac{2x}{2}=\frac{16}{2}$

$x=8$

3. $\left[(x+7)^{\frac{1}{2}}\right]^2=(x+1)^2$

$x+7=(x+1)(x+1)$

$x+7=x^2+x+x+1$

$x+7=x^2+2x+1$

$-x \; -7 \qquad -x \; -7$

$0=x^2+x-6$

$0=(x+3)(x-2)$

$0=(x+3)(x-2)$

$\boxed{x=-3 \text{ ext}}$ $\boxed{x=2}$

Check: $(-3+7)^{1/2}=-3+1$

$(4)^{1/2}=-2$

$\sqrt{4}=-2$

$2\neq -2$

Check: $(2+7)^{1/2}=2+1$

$(9)^{1/2}=3$

$\sqrt{9}=3$

$3=3$ ✓

4. $\left[(-3x-2)^{\frac{1}{3}}\right]^3 = \left[(x)^{\frac{2}{3}}\right]^3$

$-3x-2 = x^{6/3}$

$-3x-2 = x^2$

$+3x+2 \qquad +3x+2$

$0 = x^2+3x+2$

$0 = x^2+3x+2$

$0 = (x+2)(x+1)$

$\boxed{x=-2 \quad x=-1}$

5. $-3 + (8-2x)^{5/4} = 29$

$+3 \qquad +3$

$\left[(8-2x)^{5/4}\right]^{4/5} = (32)^{4/5}$

$8-2x = \sqrt[5]{32}^{\,4}$

$8-2x = (2)^4$

$8-2x = 16$

$-8 \qquad -8$

$\dfrac{-2x}{-2} = \dfrac{8}{-2}$

$\boxed{x=-4}$

Check:

$-3 + (8-2(-4))^{5/4} = 29$

$-3 + (8+8)^{5/4} = 29$

$-3 + (16)^{5/4} = 29$

$-3 + \sqrt[4]{16}^{\,5} = 29$

$-3 + (2)^5 = 29$

$-3 + 32 = 29$

$29 = 29$ ✓

32

2 16

2 8

2 4

2 2

Name____Period___ HW 15-5

ALGEBRA II UNIT 15 Simplify and Solve Rational Exponents and Roots

1. **Isolate the grouping symbol (if there is one) that contains the variable you are asked to solve for. (parenthesis, brackets, radical, numerator or denominator of a fraction)**
2. **Use inverse order of operations to isolate the needed variable.**
3. **Simplify your resulting answer if possible.**

Rewrite the equations in terms of a specific variable.

1. $A = 4\pi r^2$ (Solve for *r*)

$\frac{A}{4\pi} = \frac{4\pi r^2}{4\pi}$

$\frac{A}{4\pi} = r^2$

$\sqrt{\frac{A}{4\pi}} = r$

2. $3 \cdot V = \frac{1}{3}\sqrt{\frac{s^3}{8\pi}} \cdot 3$ (Solve for *s*)

$3V = \sqrt{\frac{s^3}{8\pi}}$

$(3V)^2 = \frac{s^3}{8\pi}$

$8\pi \cdot 9V^2 = \frac{s^3}{8\pi} \cdot 8\pi$

$72V^2\pi = s^3$

$\sqrt[3]{72V^2\pi} = s$

$2, 36, 2, 18, 2, 9, 3, 3$

$\boxed{2\sqrt[3]{9V^2\pi} = s}$

3. $\frac{9}{5} \cdot C = \frac{5}{9}(F - 32) \cdot \frac{9}{5}$ (Solve for *F*)

$\frac{9}{5}C = F - 32$

$+32 \quad +32$

$\frac{9}{5}C + 32 = F$

4. $d = \sqrt[3]{6t^2}$ (Solve for *t*)

$\frac{d^3}{6} = \frac{6t^2}{6}$

$\frac{d^3}{6} = t^2$

$\sqrt{\frac{d^3}{6}} = t \qquad d\sqrt{\frac{d}{6}} = t$

Finish problems 5-14 for homework (10 pts. each)

5. $x^3 - a^3 = 1$ (Solve for *x*)

6. $s = \frac{a}{1-r}$ (Solve for *r*)

7. $P = \frac{H}{1-r^2}$ (Solve for ***r***)

8. $d = v\sqrt{\frac{h}{4.9}}$ (Solve for ***h***)

9. $C = 100(1+r)^5$ (Solve for ***r***)

10. $r = \sqrt[3]{\frac{3V}{4\pi}}$ (Solve for ***V***)

11. $\sqrt{x^2 + a^2} = a$ (Solve for ***a***)

12. $s = \frac{10\sqrt{l}}{\sqrt{5}}$ (Solve for ***l***)

13. $V = \frac{4}{3}\pi r^3$ (Solve for ***r***)

14. $T = 2\pi\sqrt{\frac{l}{g}}$ (Solve for ***g***)

NOTES 15 - 6

Algebra II Unit 15 Simplify and Solve Rational Exponents and Roots
Application of Radicals- Real World, Area & Pythagorean Theorem

* Decimals are ok.!

Steps:
1. Determine which variables you are given values for.
2. Replace those variables with the given values.
3. Solve for the missing variable.

1. The cost of manufacturing iPhones is given by $c = 1870\sqrt{n+27}$, where c is the total cost and n is the number produced.

c represents total cost n represents number produced

What is the cost when 1 iPhone is produced? 9,895.11
$C = 1870\sqrt{1+27} = 1870\sqrt{28} =$

What is the cost when100 iPhones are produced? 11,374.77 $C = 1870\sqrt{10+27}$
$1870\sqrt{37}$

How much is that per iPhone 1,137.48 ?
$\frac{11,374.77}{10}$

What is the cost when 1,000,000 iPhone are produced? 1,870,025.25 $C = 1870\sqrt{1,000,000+27}$

How much is that per iPhone 1.87 ?
$\frac{1,870,025.25}{1,000,000}$

2. The speed in miles per hour that a motorcycle is traveling when it goes into a skid can be estimated by using the formula $s = \sqrt{18fd}$, where *f* is the coefficient of friction and *d* is the length of the skid marks in feet. After an accident, Jeff says he was going 40 miles per hour and the friction is calculated to be f = .9
S

s represents speed *f* represents friction *d* represents length of mark

How long are his skid marks 98.77 feet
$40 = \sqrt{18(.9)d}$
$1600 = 18(.9)d$
$\frac{1600}{16.2} = \frac{16.2d}{16.2}$ $d = 98.77$

What if his skid marks were 145 ft long. How fast was he really going 48.47 mph
$S = \sqrt{18(.9)(145)}$
$S = \sqrt{2349}$
$S = 48.47$

3. For a spinning amusement park ride, the velocity v in meters per second of a car moving around a curve with a radius r meters is given by $v = \sqrt{ar}$, where a is the car's acceleration in m/s^2.

v represents velocity a represents acceleration r represents radius

For safety reasons, the ride has a maximum acceleration of 39.2 m/s^2. If the cars on the ride have a velocity of 14m/s, what is the smallest radius that any curve on the ride may have 5 meters

$14 = \sqrt{39.2(r)}$

$\frac{196}{39.2} = \frac{39.2r}{39.2}$ $r = 5$

If the acceleration of a car is 8 m/s^2 while traveling around a curve with a radius of 2.5 m, what is the velocity? 4.47 m/s

$V = \sqrt{8(2.5)}$

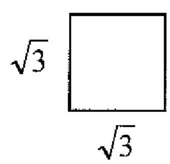
$V = \sqrt{20}$

$V = 4.47$

Write answers in radical form if necessary. **Do not write as a decimal.** Remember to simplify the radicals.

Find the Area of Each Figure

1.



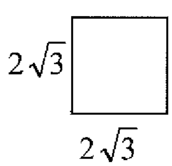
$A = l \cdot w$

$A = \sqrt{3} \cdot \sqrt{3}$

$= \sqrt{9}$

$= 3$

2.



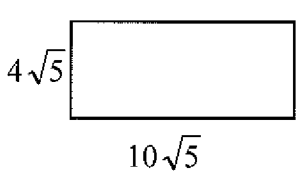
$2\sqrt{3} \cdot 2\sqrt{3}$

$4\sqrt{9}$

$2 \cdot 3$

6

3.



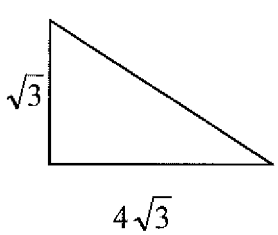
$4\sqrt{5} \cdot 10\sqrt{5}$

$40\sqrt{25}$

$40 \cdot 5$

200

4.



$A = \frac{1}{2}bh$

$\frac{1}{2} \cdot 4\sqrt{3} \cdot \sqrt{3}$

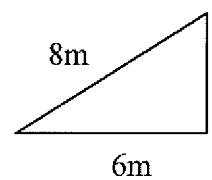
$\frac{1}{2} \cdot 4\sqrt{9}$

$\frac{1}{2} \cdot 4 \cdot 3$

6

Find the Missing Side of each Right Triangle

7.



$a^2 + b^2 = c^2$

$a^2 + 6^2 = 8^2$

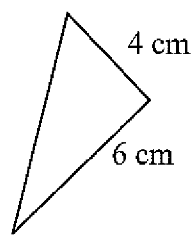
$a^2 + 36 = 64$

$a^2 = 28$

$a = \sqrt{28} = 2\sqrt{7}$

28 — 2, 14 — 2, 7

8.



$4^2 + 6^2 = c^2$

$16 + 36 = c^2$

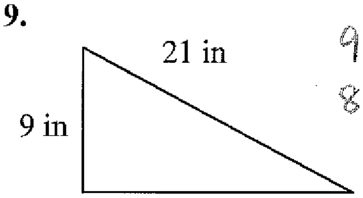
$52 = c^2$

$\sqrt{52} = c$

$2\sqrt{13} = c$

52 — 2, 26 — 2, 13

9.



$9^2 + b^2 = 21^2$

$81 + b^2 = 441$

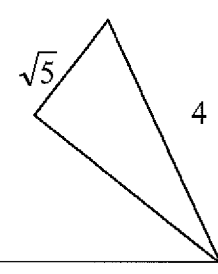
$b^2 = 360$

$b = \sqrt{360}$

$b = 6\sqrt{10}$

360 — 2, 180 — 2, 90 — 2, 45 — 3, 15 — 3, 5

10.



$(\sqrt{5})^2 + b^2 = 4^2$

$5 + b^2 = 16$

$b^2 = 11$

$b = \sqrt{11}$