

NOTES 14 - 1

ALGEBRA II UNIT 14 Add, Subtract and Solve Rational Expressions

REMEMBER:

FOIL: $(x+2)(x-5)$

$$x^2 - 5x + 2x - 10$$

$$x^2 - 3x - 10$$

Subtracting a polynomial: $x + 6 - (3x^2 - 4x - 8)$

$$\begin{array}{r} x + 6 \\ - (3x^2 - 4x - 8) \\ \hline -3x^2 + 4x + 8 \end{array}$$

$$-3x^2 + 5x + 14$$

What must fractions have in order to **add or subtract** them?

Common denominator

What must we do first to determine a common denominator?

- Factor all denominators, if possible
- Use one of each factor to create the common denominator

How do we create equivalent fractions so they all have a common denominator?

Multiply all terms by the factors missing in their denominators
[both numerator & denominator]

What if you already had a common denominator? Ready to add or subtract!

$$\frac{u-v}{8v} + \frac{6u-3v}{8v}$$

$$\frac{7u-4v}{8v}$$

$$\frac{x+2}{2x^2+13x+20} - \frac{2x+7}{2x^2+13x+20}$$

$$\frac{-x-5}{2x^2+13x+20}$$

Factor the denominator and find a common denominator before you begin.

$$\frac{\overbrace{(x+4)}^{\curvearrowright} x}{\overbrace{(x+4)}^{\curvearrowright} x - 4} + \frac{-30}{x^2 - 16}$$

$$\frac{(x+4)(x-4)}{(x+4)(x-4)}$$

$$\frac{6}{x^2 + 4x - 32} + \frac{\overbrace{(x-5)}^{\curvearrowright} \overbrace{(x+8)}^{\curvearrowright}}{x-4(x+8)}$$

$$\frac{(x+8)(x-4)}{(x+8)(x-4)}$$

CD:

$$\frac{(x+4)(x-4)}{(x+4)(x-4)} \cdot \frac{x^2 + 4x + -30}{(x+4)(x-4)}$$

CD:

$$\frac{6 + x^2 + 8x - 5x - 40}{(x+8)(x-4)}$$

$$\frac{x^2 + 3x - 34}{(x+8)(x-4)}$$

$$\frac{\overbrace{(x+7)}^{\curvearrowright} x}{x^2 - 5x - 14} + \frac{2x}{\overbrace{x^2 + 9x + 14}^{\curvearrowright}}$$

$$\frac{(x+7)(x-7)(x+2)}{(x+7)(x-7)(x+2)} \cdot \frac{(x+7)(x+2)(x-7)}{(x+7)(x+2)(x-7)}$$

$$\frac{\overbrace{(x-8)}^{\curvearrowright} 3}{(x-8)x+7} + \frac{4}{\overbrace{x-8}^{\curvearrowright} \overbrace{(x+7)}^{\curvearrowright}}$$

$$\frac{(x-8)x+7}{(x-8)(x+7)}$$

CD:

$$(x-7)(x+2)(x+7) \cdot \frac{x^2 + 7x - 2x^2 + 14x}{(x+7)(x-7)(x+2)}$$

$$\frac{-x^2 + 21x}{(x+7)(x-7)(x+2)}$$

CD:

$$\frac{3x - 24 - 4x - 28}{(x-8)(x+7)}$$

$$\frac{-x - 52}{(x-8)(x+7)}$$

OYO :

$$\frac{5}{n+5} + \frac{4n}{n+6}$$

$$\frac{y^3}{y^3} \cdot \frac{11x^2}{12x^3y^2} + \frac{7y^3}{4xy^5} \cdot \frac{3x^2}{3x^2}$$

$$\frac{11x^2y^3 + 21x^2y^3}{12x^3y^5}$$

CD:

$$12x^3y^5$$

$$\frac{32x^2y^3}{12x^3y^5}$$

$$\frac{8}{3xy^2}$$

NOTES 14 - 2

ALGEBRA II UNIT 14 Solve Rational Equations and Graph Rational Functions

Solving Rational Equations: Solve each rational equation. Check for extraneous solutions/zero denominators.

Factor, simplify, solve for x.

$$1. \frac{(x+8)(x-8)}{x-8} = 5 \quad \begin{array}{l} x+8 = 5 \\ -8 \quad -8 \end{array}$$

$$\boxed{x = -3}$$

$$-3-8 \neq 0 \checkmark$$

$$2. \frac{(x+3)(x+3)}{x+3} = 6 \quad \begin{array}{l} x+3 = 6 \\ -3 \quad -3 \end{array}$$

$$\boxed{x = 3}$$

$$3+3 \neq 0 \checkmark$$

What do we do when factors don't cancel out like above?

1. Factor all denominators
2. Determine a common denominator for ALL terms
3. Create equivalent fractions with the common denominator
4. Solve the numerator for x * check for extraneous solutions

$$3. \frac{(x+6)-4}{(x+6)x+4} = \frac{-3}{x+6} \frac{(x+4)}{(x+4)}$$

$$\begin{array}{r} -4x-24 = -3x-12 \\ +4x \quad \quad +4x \end{array}$$

$$\begin{array}{r} -24 = x-12 \\ +12 \quad +12 \end{array}$$

$$\boxed{-12 = x}$$

$$4. \frac{2(x-5)}{2(x-2)} + \frac{x(x-2)}{2(x-2)} = \frac{3}{x-2} (2)$$

$$2x-10 + x^2-2x = 6$$

$$\begin{array}{r} x^2-10 = 6 \\ +10 \quad +10 \end{array}$$

$$\sqrt{x^2} = \sqrt{16}$$

$$\boxed{x = 4}$$

Common Denominator: (x+4)(x+6)

$$-12+4 \neq 0 \checkmark$$

$$-12+6 \neq 0 \checkmark$$

Common Denominator: 2(x-2)

$$4-2 \neq 0 \checkmark$$

$$5. \quad \frac{16}{x^2 - 25} = \frac{2}{x-5} \frac{(x+5)}{(x+5)(x-5)}$$

$$16 = \frac{2x + 10}{-10} \frac{-10}{-10}$$

$$\frac{6}{2} = \frac{2x}{2} \quad [X=3]$$

$$6. \quad \frac{(x+1) 5 \cdot 2(x-1) 2}{(x+1)x-1 \cdot x+1} = \frac{2}{x^2-1}$$

$$(x+1)(x-1)(x-1)$$

$$5x+5 - 2x+2 = 2$$

$$3x + 7 = 2$$

$$-7 \quad -7$$

$$\frac{3x}{3} = \frac{-5}{3}$$

$$[X = -1.67]$$

$$7. \quad \frac{x-3}{2x^2-13x-7} \frac{1(x-7)}{2x+1} = \frac{4}{x-7} \frac{(2x+1)}{(2x+1)(x-7)}$$

$$x-3 - x+7 = 8x+4$$

$$4 = 8x+4$$

$$-4 \quad -4$$

$$\frac{0}{8} = \frac{8x}{8}$$

$$[X=0]$$

$$8. \quad \frac{2(x+4)}{2(x-6)} + \frac{x(x-6) 10}{2(x-6)} = \frac{20}{x-6}$$

$$2x+8 + x^2-6x = 20$$

$$x^2 - 4x + 8 = 20$$

$$20 \quad -20$$

$$x^2 - 4x - 12 = 0$$

$$(x-6)(x+2) = 0$$

$$[X=6 \quad X=-2]$$

$$\text{Common Denominator: } \underline{(x+5)(x-5)}$$

$$3^2 - 25 \neq 0 \quad \checkmark$$

$$3-5 \neq 0 \quad \checkmark$$

$$\text{Common Denominator: } \underline{(x+1)(x-1)}$$

$$-1.67-1 \neq 0 \quad \checkmark$$

$$-1.67+1 \neq 0 \quad \checkmark$$

$$(-1.67)^2 - 1 \neq 0 \quad \checkmark$$

$$\text{Common Denominator: } \underline{(2x+1)(x-7)}$$

$$2(0)^2 - 13(0) - 7 \neq 0 \quad \checkmark$$

$$2(0)+1 \neq 0 \quad \checkmark$$

$$0-7 \neq 0 \quad \checkmark$$

$$\text{Common Denominator: } \underline{2(x-6)}$$

$$\text{Extraneous Solution: } \underline{X=6}$$

$$6-6 = 0 \quad \checkmark$$

$$-2-6 \neq 0 \quad \checkmark$$

$$\begin{array}{r} -12 \quad -4 \\ x \overline{) 16} \\ \underline{-6 \cdot 2} \quad -6+2 \end{array}$$

NOTES 14 - 3

ALGEBRA II Unit 14 Simplify and Solve Rationals

Use direct and inverse variation to set up and solve the following functions.

DIRECT VARIATION:

As x increases, what happens to y ?

y also increases

As x decreases, what happens to y ?

y also decreases

Formula: $y = Kx$

1. If y varies **directly** with x , and $y=16$ when $x=10$, find y when $x=13$.

$$y = Kx$$
$$\frac{16}{10} = \frac{K(10)}{10}$$

$$K = 1.6$$

$$y = 1.6x$$
$$y = 1.6(13)$$

$$y = 20.8$$

2. A 2000 pound shark is caught off the coast off Catalina Island. It was measured at 16 feet long. The weight **varies directly** with the length. How heavy would the 25 foot long shark in Jaws be?

$$W = Kl$$
$$\frac{2000}{16} = \frac{K(16)}{16}$$

$$K = 125$$

$$W = 125l$$
$$W = 125(25)$$

$$W = 3125 \text{ pounds}$$

3. The number of houses that can be served by a water main is **directly proportional** to the diameter of the main. Suppose that City Waterworks has a 10 cm diameter water main that can supply 50 houses with water. How many houses could be served if the diameter was 30 cm?

$$H = kd$$
$$\frac{50}{10} = \frac{K(10)}{10}$$

$$K = 5$$

$$H = 5d$$
$$H = 5(30)$$

$$H = 150 \text{ houses}$$

INVERSE VARIATION:

As x increases, what happens to y ?

y decreases

As x decreases, what happens to y ?

y increases

Formula: $y = \frac{K}{x}$

4. If y varies **inversely** with x , and $y=12$ when $x=7$, find x when $y=13$.

$$y = \frac{K}{x}$$
$$7 \cdot 12 = \frac{K}{7} \quad K = 84$$
$$y = \frac{84}{x} \quad x \cdot 13 = \frac{84}{x} \cdot x$$
$$\frac{13x}{13} = \frac{84}{13} \quad x = 6.46$$

5. The current I in an electrical conductor **varies inversely** as the resistance R of the conductor. The current is 3 amperes when the resistance is 516 ohms. What is the current when the resistance is 332 ohms?

$$I = \frac{K}{R}$$
$$516 \cdot 3 = \frac{K}{516} \cdot 516 \quad K = 1548$$
$$I = \frac{1548}{R}$$
$$I = \frac{1548}{332} \quad I = 4.66 \text{ amperes}$$

6. The amount of force you must exert on a wrench handle to loosen a rusty bolt is **inversely proportional** to the length of the handle. A 7 inch wrench requires 270 pounds of force. How long would the wrench need to be if you can only exert 300 pounds of force?

$$F = \frac{K}{l}$$
$$7 \cdot 270 = \frac{K}{7} \cdot 7 \quad K = 1890$$
$$F = \frac{1890}{l}$$
$$l \cdot 300 = \frac{1890}{300} \cdot l$$
$$\frac{300 \cdot l}{300} = \frac{1890}{300} \quad l = 6.3 \text{ inches}$$

7. One author claimed that the amount of time a business spends discussing an item in the budget is **inversely proportional** to the amount of money involved. If a university spends 15 minutes discussing a \$1,000,000 computer upgrade, how much time would they spend discussing \$1,000 bicycle rack?

$$T = \frac{K}{B}$$
$$1,000,000 \cdot 15 = \frac{K}{1,000,000} \cdot 1,000,000 \quad 15,000,000 = K$$
$$T = \frac{15,000,000}{B}$$
$$T = \frac{15,000,000}{1,000} \quad T = 15,000 \text{ minutes}$$