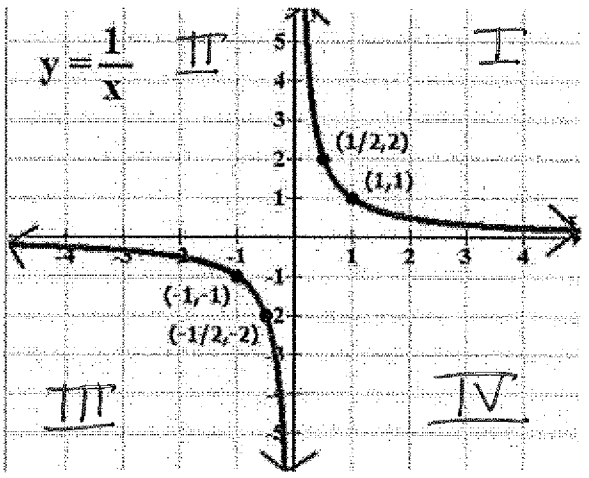
NOTES 12-1

Algebra II UNIT 12 Graphing Rational Functions
Domain, Range, and Transformations
Parent Function:

Name: Rational

Equation: $y = \frac{1}{x}$ Graph



Asymptotes: where the graph approaches but never comes to

Vertical Asymptote: $x = 0$ → interrupts the domain

Horizontal Asymptote: $y = 0$ → interrupts the range

Domain: $(-\infty, 0) \cup (0, \infty)$ or $x \neq 0$

Range: $(-\infty, 0) \cup (0, \infty)$ or $y \neq 0$

Transformations: $y = \frac{a}{b(x-h)} + k$

a = vertical stretch, vertical compression, vertical reflection
b= horizontal stretch, horizontal compression, horizontal reflection
h = horizontal shift left or right
k = vertical shift up or down

Describe the Transformations:

$y = \frac{3}{x+3} - 1$

VA → horizontal shift
HA → vertical shift

1. V. Stretch of 3
2. Left 3
3. Down 1

v. Asymptote $x = -3$
h. Asymptote $y = -1$
Domain $x \neq -3$ or $(-\infty, -3) \cup (-3, \infty)$
Range $y \neq -1$ or $(-\infty, -1) \cup (-1, \infty)$

2. $y = \frac{-1}{(x-5)} + 3$

1. V. Reflection
2. Right 5
3. Up 3

v. Asymptote $x = 5$

h. Asymptote $y = 3$

Domain $(-\infty, 5) \cup (5, \infty)$ or $x \neq 5$

Range $(-\infty, 3) \cup (3, \infty)$ or $y \neq 3$

3. $y = \frac{1}{x-2} + 5$

1. Right 2
2. Up 5

v. Asymptote $x = 2$

h. Asymptote $y = 5$

Domain $(-\infty, 2) \cup (2, \infty)$ $x \neq 2$

Range $(-\infty, 5) \cup (5, \infty)$ $y \neq 5$

4. $y = \frac{-2}{x+4} - 5$

1. V. Reflection
2. V. Stretch of 2
3. Left 4
4. Down 5

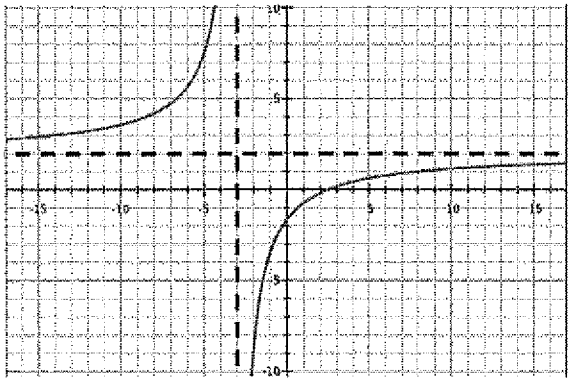
v. Asymptote $x = -4$

h. Asymptote $y = -5$

Domain $(-\infty, -4) \cup (-4, \infty)$ $x \neq -4$

Range $(-\infty, -5) \cup (-5, \infty)$ $y \neq -5$

Given the graph:



Transformations:

1. Left 3
2. Up 2
3. V. Reflection

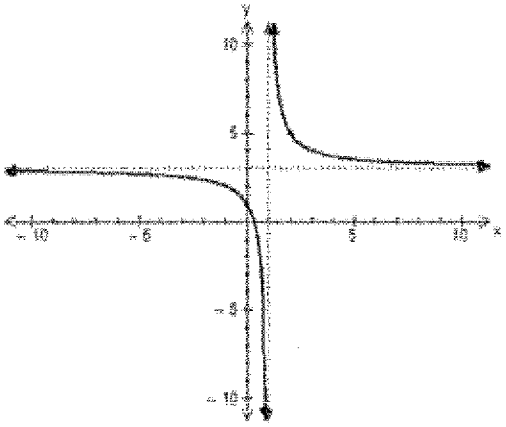
v. Asymptote $x = -3$

h. Asymptote $y = 2$

Domain $(-\infty, -3) \cup (-3, \infty)$ $x \neq -3$

Range $(-\infty, 2) \cup (2, \infty)$ $y \neq 2$

Equation: $y = \frac{-1}{x+3} + 2$



Transformations:

1. Right 2
2. Up 3

v. Asymptote $x = 2$

h. Asymptote $y = 3$

Domain $(-\infty, 2) \cup (2, \infty)$ $x \neq 2$

Range $(-\infty, 3) \cup (3, \infty)$ $y \neq 3$

Equation: $y = \frac{1}{x-2} + 3$

NOTES 12-2

Algebra II UNIT 12 Graphing Rational Functions

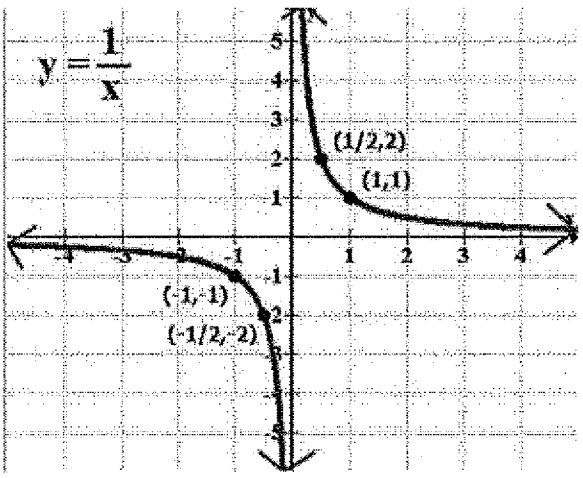
Graphing Parent Function Transformations

Parent Function:

Name: Rational

Equation: $y = \frac{1}{x}$

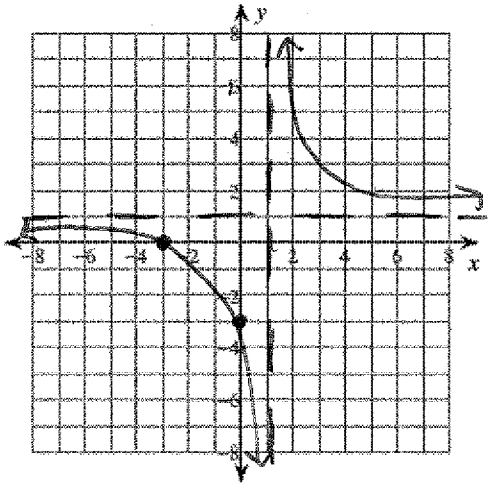
Graph



Describe the Transformations:

$y = \frac{4}{x-1} + 1$

1. V. Stretch of 4
2. Right 1
3. Up 1



v. Asymptote $x = 1$

h. Asymptote $y = 1$

x-intercept: (-3, 0)

$0 = \frac{4}{x-1} + 1$

$-1 \quad\quad -1$

$(x-1)\cdot -1 = \frac{4}{x-1}\cdot(x-1)$

$\frac{-1(x-1)}{-1} = \frac{4}{-1}$

$x - 1 = -4$

$+1 \quad +1$

$x = -3$

y-intercept: (0, -3)

$y = \frac{4}{0-1} + 1$

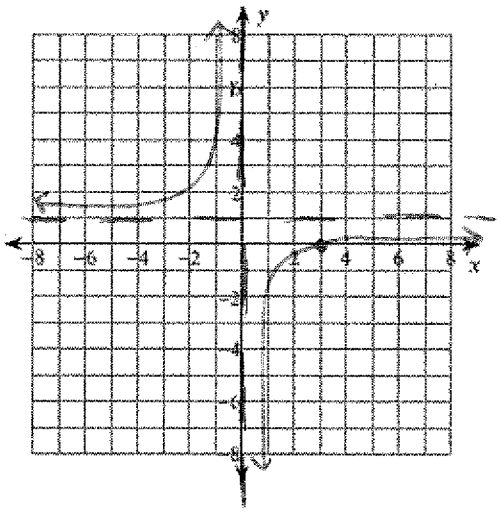
$y = \frac{4}{-1} + 1$

$y = -4 + 1$

$y = -3$

Domain $(-\infty, 1) \cup (1, \infty)$ $x \neq 1$ Range $(-\infty, 1) \cup (1, \infty)$ $y \neq 1$

2. $y = \frac{-3}{x} + 1$



1. V. Reflection
2. V. Stretch of 3
3. Up 1

v. Asymptote $x = 0$

h. Asymptote $y = 1$

x-intercept: (3, 0)

$0 = \frac{-3}{x} + 1$

$-1 \quad -1$

$x \cdot -1 = \frac{-3}{x} \cdot x$

$\frac{-x}{-1} = \frac{-3}{-1}$

$x = 3$

y-intercept: (~~0,~~) none

$y = \frac{-3}{0} + 1$

Undefined

Domain $(-\infty, 0) \cup (0, \infty)$ $x \neq 0$ Range $(-\infty, 1) \cup (1, \infty)$ $y \neq 1$

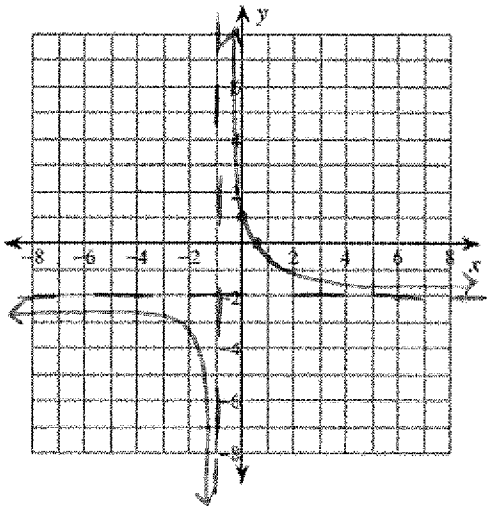
3. $y = \frac{3}{x+1} - 2$

1. V. Stretch of 3 3. Down 2
2. Left 1

v. Asymptote $x = -1$

h. Asymptote $y = -2$

x-intercept: (.5, 0)



$0 = \frac{3}{x+1} - 2$

$+2 \quad +2$

$(x+1)\, 2 = \frac{3}{x+1}(x+1)$

$\frac{2(x+1)}{2} = \frac{3}{2}$

$x + 1 = \frac{3}{2}$

$-1 \quad -1$

$x = .5$

y-intercept: (0 , 1)

$y = \frac{3}{0+1} - 2$

$y = \frac{3}{1} - 2$

$y = 3 - 2$

$y = 1$

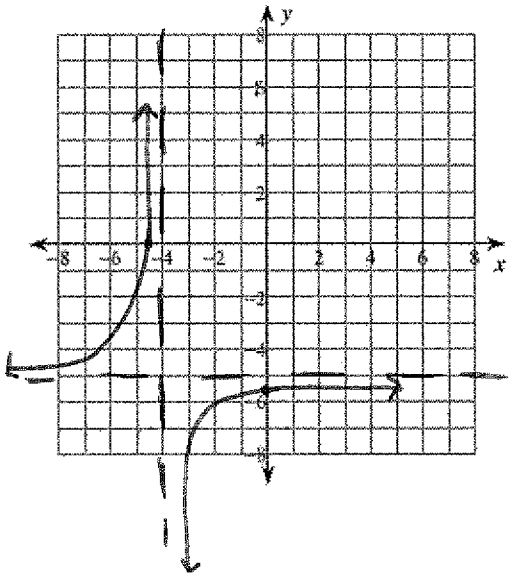
Domain $(-\infty, -1) \cup (-1, \infty)$ $x \neq -1$ Range $(-\infty, -2) \cup (-2, \infty)$ $y \neq -2$

4. $y = \frac{-2}{x+4} - 5$

1. V. Reflection
2. V. Stretch of 2
3. Left 4
4. Down 5

v. Asymptote $x = -4$

h. Asymptote $y = -5$



x-intercept: $(-4.4, 0)$

$0 = \frac{-2}{x+4} - 5$

$+5 \quad +5$

$(x+4)5 = \frac{-2}{x+4}(x+4)$

$\frac{5(x+4)}{5} = \frac{-2}{5}$

$x + 4 = \frac{-2}{5}$

$-4 \quad -4$

$x = -\frac{22}{5} = -4.4$

y-intercept: $(0, -5.5)$

$y = \frac{-2}{0+4} - 5$

$y = \frac{-2}{4} - 5$

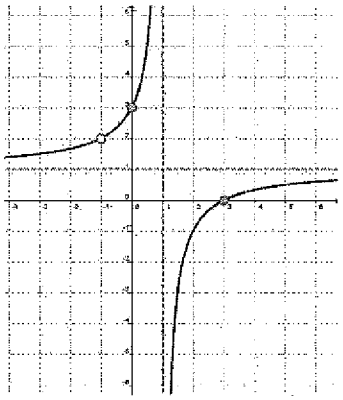
$y = -\frac{1}{2} - 5$

$y = -\frac{11}{2} = -5.5$

Domain $(-\infty, -4.4) \cup (-4.4, \infty)$ $x \neq -4.4$ Range $(-\infty, -5.5) \cup (-5.5, \infty)$ $y \neq -5.5$

NOTES 12-3

Algebra II UNIT 12 Graphing Rational Functions
Graphing Parent Function Transformations



Discontinuities:

Vertical Asymptotes: found where a value would create a zero denominator

Holes: when the same value that would create a zero denominator is also in the numerator

Example 1: $y = \frac{x+1}{x-2}$

$x-2=0$

VA at $x=2$

Example 2: $y = \frac{(x+1)(x-2)}{(x-2)}$

$x-2=0$

Hole at $x=2$

Plug $x=2$ into the equation to find y

$y = x+1$

$y = 2+1$

$y = 3$

$(2,3)$

Horizontal Asymptotes: are determined by the degree of the numerator and denominator are compared

Example 1: $y = \frac{x+1}{x^2-2}$ $\quad \frac{\text{zillion}+1}{(\text{zillion})^2-2} \rightarrow$ insignificant $\quad \frac{\text{large}}{\text{much larger}} \rightarrow$ super small fraction, leads to zero

* larger degree in denominator → HA $y=0$

Example 2: $y = \frac{3x+1}{2x-2}$ $\quad \frac{3x}{2x} \rightarrow$ coefficients will remain

* same degrees → HA $y = \frac{3}{2}$

Example 3: $y = \frac{x^2+1}{x-2}$ $\quad \frac{(\text{zillion})^2+1}{\text{zillion}-2} \rightarrow$ insignificant $\quad \frac{\text{much larger}}{\text{large}} \rightarrow$ won't lead to zero

* larger degree in numerator → HA none

* will have a slant asymptote

Practice Problems:

1. $f(x) = \frac{x^2+7x+6}{x+3}$ $(x+6)(x+1)$

6	7
x	+
6·1	6+1

Vertical Asymptote: none Holes: none

Horizontal Asymptote: none

Roots: $(-6,0)$ $(-1,0)$

Domain: $(-\infty,\infty)$ Range: $(-\infty,\infty)$

2. $f(x) = \frac{x^2+x-6}{x-2}$ $(x+3)(x-2)$

-6	1
x	+
3·-2	3+-2

$x-2=0$
$+2 \quad +2$
$x=2$

Vertical Asymptote: none Holes: $x=2$

Horizontal Asymptote: none

Roots $(-3,0)$

Domain: $(-\infty,2)\cup(2,\infty)$ Range: $(-\infty,5)\cup(5,\infty)$

Plug in to find y value
$(x+3)$
$(2+3)=5$

3. $f(x) = \frac{x-2}{x^2+x}$ $x(x+1)$

Vertical Asymptote: $x=0$ $x=-1$ Holes: none

Horizontal Asymptote: $y=0$

Roots: $(2,0)$

Domain: $(-\infty,-1)\cup(-1,0)\cup(0,\infty)$ Range: $(-\infty,0)\cup(0,\infty)$

4. $f(x) = \frac{x^2+2x-15}{x-1}$ $(x+5)(x-3)$

-15	2
x	+
5·-3	5+-3

Vertical Asymptote: $x=1$ Holes: none

Horizontal Asymptote: none

Roots: $(-5,0)$ $(3,0)$

Domain: $(-\infty,1)\cup(1,\infty)$ Range: $(-\infty,\infty)$

5. $f(x) = \frac{3x^2+8}{2x^2+1}$

Vertical Asymptote: none Holes: none

Horizontal Asymptote: $y=3/2$

Domain: $(-\infty,\infty)$ Range: $(-\infty,3/2)\cup(3/2,\infty)$