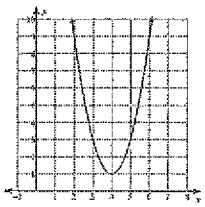
NOTES 11 - 1

ALGEBRA II UNIT 11 Graphing Higher Order Polynomials

Even DEGREE FUNCTIONS: Ends go in the Same direction **vs.** **Odd** DEGREE FUNCTIONS: Ends go in opposite directions

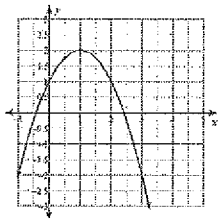
even
$y = 2x^2 - 16x + 33$



Left As x → −∞ y→ ∞

Right As x → ∞ y→ ∞

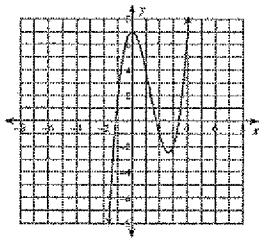
even
$y = -x^2 + 2x + 1$



As x → −∞ y→ −∞

As x → ∞ y→ −∞

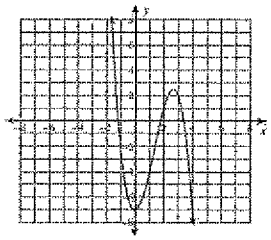
odd
$f(x) = x^3 - 4x^2 + 7$



As x → −∞ f(x)→ −∞

As x → ∞ f(x)→ ∞

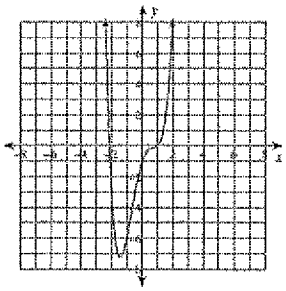
odd
$f(x) = -x^3 + 4x^2 - 7$



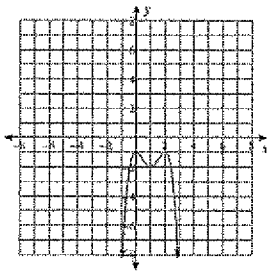
As x → −∞ f(x)→ ∞

As x → ∞ f(x)→ −∞

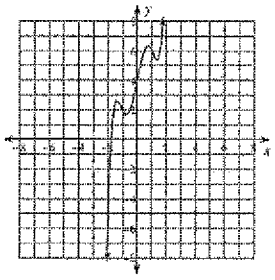
$f(x) = x^4 - 3x^2 + 3x - 1$



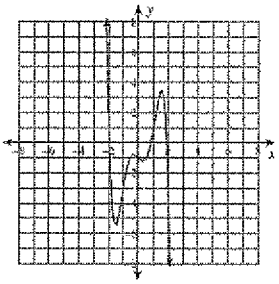
$f(x) = -x^4 + 4x^3 - 4x^2 - 1$



$f(x) = x^5 - 4x^3 + 5x + 4$

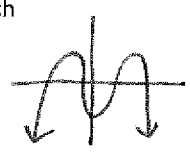


$f(x) = -x^5 + 4x^3 - x - 1$



$x \to -\infty \quad f(x) \to$

$x \to \infty \quad f(x) \to$

$f(x) = -x^4 + x^2 + 2$	$f(x) = 2x^2 - 3$	$f(x) = x^3 + 10x^2 + 32x + 34$	$f(x) = x^5 - 4x^3 + x + 1$
Even or odd?	Even or odd?	Even or odd?	Even or odd?
Positive or negative?	Positive or negative?	Positive or negative?	Positive or negative?
Sketch 	Sketch 	Sketch 	Sketch 
End Behavior	End Behavior	End Behavior	End Behavior
As $x \to -\infty$ $f(x) \to -\infty$	$x \to -\infty$ $f(x) \to \infty$	$x \to -\infty$ $f(x) \to -\infty$	$x \to -\infty$ $f(x) \to -\infty$
As $x \to \infty$ $f(x) \to -\infty$	$x \to \infty$ $f(x) \to \infty$	$x \to \infty$ $f(x) \to \infty$	$x \to \infty$ $f(x) \to \infty$

Roots & Multiplicity

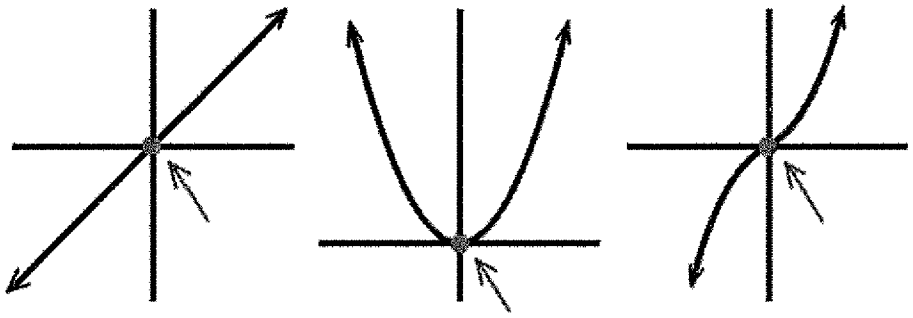
Multiplicity refers to the number of times a factor occurs in the factorization of a polynomial.

A. If a root is singular(*multiplicity* 1), it **passes** through the **x**-axis. (like a line)

B. If a root is a double root *(multiplicity 2)*, it will **bounce** when it hits the x-axis.

C. If a root is triple *(multiplicity 3)*, it **curves** like a cubic.

$f(x) = x$ $f(x) = x^2$ $f(x) = x^3$



$(x-2)^3(x-3)^2(x+4)$

$x = -4$ has a multiplicity of 1

$x = 3$ has a multiplicity of 2

$x = 2$ has a multiplicity of 3.

EXAMPLE 1

x= -15 with multiplicity 1

x= -10 with multiplicity 2

x= -5 with multiplicity 1

x= 0 with multiplicity 1

x= 10 with multiplicity 2

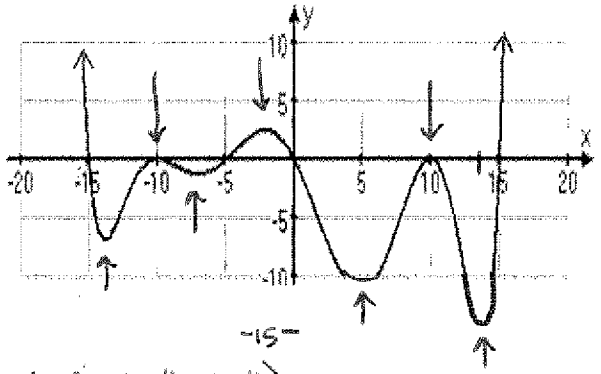
x= 15 with multiplicity 1

odd or even Degree: even (same direction) Leading coefficient + or - + ends up

Domain: $(-\infty, \infty)$ Range: $[-15, \infty)$

End Behavior As $x \to -\infty$, $y \to \infty$

As $x \to \infty$, $y \to \infty$



Does it have an absolute max? No

Why? continues up infinitely

Does it have an absolute min? Yes (13, -15)

it's THE lowest point on the graph

-15-

Does it have a local max? Yes How many? 3 (all of the "hills")

Does it have a local min? Yes How many? 4 (all of the "valleys")

Factors: $x(x+15)(x+10)^2(x+5)(x-10)^2(x-15)$

$x = -15 \to (x+15)$

What is the degree of the polynomial that represents this graph? 8, add all multiplicities

EXAMPLE 2

x= -3 with multiplicity 1 odd or even Degree: odd (ends opposite) Leading coefficient + or - + ends up

x= -2 with multiplicity 1 Domain: $(-\infty, \infty)$

x= 1 with multiplicity 3 Range: $(-\infty, \infty)$

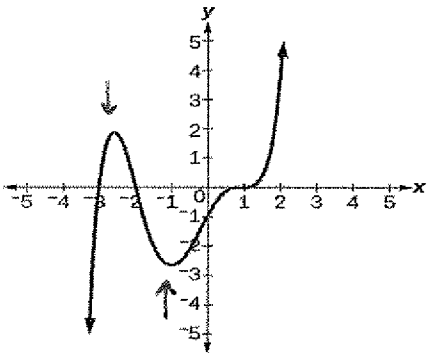
End Behavior As $x \to -\infty$, $y \to -\infty$ and As $x \to \infty$, $y \to \infty$

Does it have an absolute max? No

Why? Continues to $+\infty$

Does it have an absolute min? No

Why? Continues to $-\infty$



Does it have a local max? Yes How many? 1

Does it have a local min? Yes How many? 1

Factors: $(x+3)(x+2)(x-1)^3$

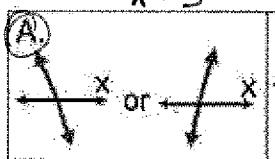
What is the degree of the graphed polynomial? 5

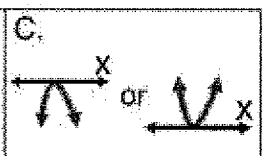
On Your Own:

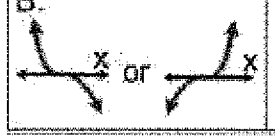
1. How does f(x) intersect the x-axis at 3?

$f(x) = 7(x-3)^1(5x-4)^3(x+7)^2$

$x = 3$

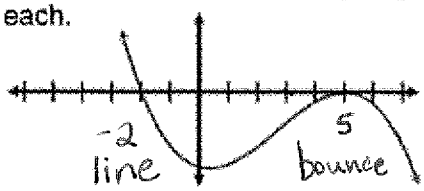
A. 

C. 

B. 

D. This polynomial function does not intercept the x axis at this point.

2. Find the zeros of the polynomial function and state the multiplicity of each.



A. -2 multiplicity 2; 5 multiplicity 1
B. 2 multiplicity 1; -5 multiplicity 2
(C.) -2 multiplicity 1; 5 multiplicity 2
D. 2 multiplicity 2; -5 multiplicity 1

3. How does f(x) intersect the x-axis at $\frac{1}{4}$?

$f(x) = (2x-3)(4x-1)^3(3x+7)^2$

$4x-1=0$
$+1 \quad +1$
$\frac{4x}{4} = \frac{1}{4}$
$x = \frac{1}{4}$

A. Pass
B. Bounce
(C.) Curve
D. Skip

$-x = 0$
$\frac{-x}{-1} = \frac{0}{-1}$
$x = 0$ mult 1

$2x-3=0$
$\frac{2x}{2} = \frac{3}{2}$
$x = 3/2$ mult. 1

4. Find the zeros of the polynomial function and state the multiplicity of each.

$y = -x(2x-3)(5x+1)^3$

(A.) 0 multiplicity 1; 3/2 multiplicity 1; -1/5 multiplicity 3
B. 3/2 multiplicity 1; 1/5 multiplicity 3
C. 0 multiplicity 1; 2/3 multiplicity 1; -1/5 multiplicity 3
D. 2/3 multiplicity 1; -1/5 multiplicity 3

$5x+1=0$
$5x=-1$
$x = -1/5$ mult. 3

ALGEBRA II UNIT 11 Graphing Higher Order Polynomials

NOTES 11-2

Given the equation: $-x^4+3x^3+13x^2-27x-36=0$. (Leading → degree)

How many solutions are there? 4 * look at degree

Is the degree of the polynomial even or odd? even (same direction)

Is the leading coefficient positive or negative? negative (ends down)

Do synthetic division with the following solutions. Remember to look if place holders are needed.

x = -1, x = 3 ← roots not factors, so no need to solve / change signs

-1	-1	3	13	-27	-36
	↓	1	-4	-9	36
3	-1	4	9	-36	0
	↓	-3	3	36	
	$-1x^2$	$1x$	12	0	

List the **2 linear factors** and the **1 quadratic factor** you just found: $(x+1)(x-3)(-x^2+x+12)$

Factor the quadratic: (Grouping or short-cut)

***Factor out -1 if the leading coefficient is negative.**

because $a=1$

$$\frac{-x^2+x+12}{-1}$$

$-1(x^2-x-12)$

$-1(x-4)(x+3)$

$-12 \mid -1$

$x \mid +$

$-4 \cdot 3 \mid -4+3$

List all **4** linear factors with the sign of the leading coefficient: $-1(x-4)(x+3)(x+1)(x-3)$ (Given)

SOLVE TO FIND ALL 4 roots/zeros/solutions/x-intercepts

$x-4=0$ → $x=4$

$x+3=0$ → $x=-3$

$x+1=0$ → $x=-1$

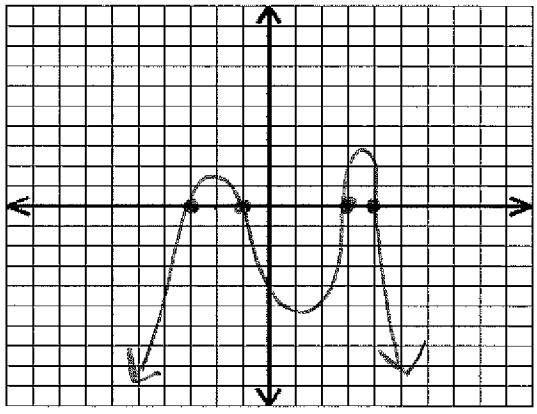
$x-3=0$ → $x=3$

List all 4 solutions/zeros: $x=4,\ x=-3,\ x=-1,\ x=3$

1. Describe the **end behavior**
   (*Think about degree and leading coefficient)

   As $x \to -\infty,\ y \to -\infty$

   As $x \to \infty,\ y \to -\infty$
2. **Plot** all roots on the graph
3. Sketch the **graph** from left to right.



Is there an absolute maximum? Yes

Is there an absolute minimum? No

Domain $(-\infty, \infty)$ Range $(-\infty, \text{Max}]$

Given the equation: How many solutions are there? $6x^5 - 19x^4 - 143x^3 - 138x^2 + 76x + 56 = 0$

How many solutions are there 5

Is the degree of the polynomial even or odd odd (opposite directions)

Is the leading coefficient positive or negative positive (ends up)

Do synthetic division with the following solutions. Remember to look if place holders are needed.

x = -2 multiplicity of 2, x=7

mult. of 2 means 2 of these roots

-2	6	-19	-143	-138	76	56
	↓	-12	62	162	-48	-56
-2	6	-31	-81	24	28	0
	↓	-12	86	-10	-28	
7	6	-43	5	14	0	
	↓	42	-7	-14		
	$6x^2$	$-1x$	-2	0		

List the **3 linear** and **1 quadratic** factors: $(x+2)^2(x-7)(6x^2 - x - 2)$

Factor the quadratic: (Grouping or short-cut)

a > 1

***Factor out -1 if the leading coefficient is negative.**

$6x^2 - x - 2$

$6x^2 - 4x + 3x - 2$

$2x \quad 1$

$-12 \quad -1$

$-4 \cdot 3 \quad -4+3$

$2x(3x-2) + 1(3x-2)$

$(3x-2)(2x+1)$

List all **5** linear factors with the sign of the leading coefficient: $(x+2)^2(x-7)(3x-2)(2x+1)$

SOLVE TO FIND ALL 5 roots/zeros/solutions/x-intercepts

$x+2=0$ $\quad x=-2$

$x-7=0$ $\quad x=7$

$3x-2=0$ $\quad 3x=2$ $\quad x=2/3$

$2x+1=0$ $\quad 2x=-1$ $\quad x=-1/2$

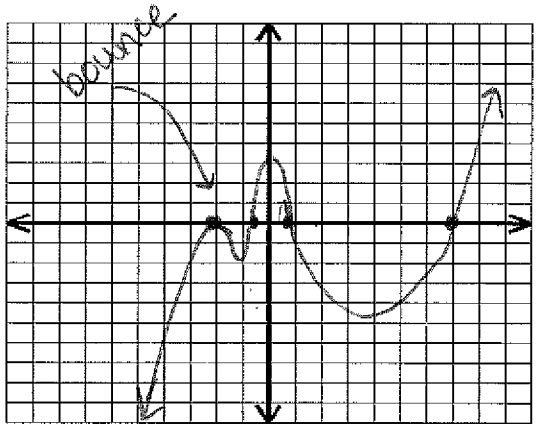
List all 5 solutions/zeros: $x=-2$ mult. of 2, $x=7$, $x=2/3$, $x=-1/2$

4. Describe the **end behavior**
   (*Think about degree and leading coefficient)

   As $x \to -\infty$, $y \to -\infty$

   As $x \to \infty$, $y \to \infty$

5. **Plot** all roots on the graph
6. Sketch the **graph** from left to right.



Is there an absolute maximum? No

Is there an absolute minimum? No

Domain $(-\infty, \infty)$ Range $(-\infty, \infty)$