Notes 10-1

ALGEBRA II UNIT 10 Polynomial Operations

Adding Polynomials

- Find like terms and combine.
- Like terms have the same variables raised to the same exponents

1) $(2x^2 + 3x + 2) + (x^2 - 5x - 5)$	2) $(3a^3 + 3ab - b^2) + (6b^2 + 4ab)$	3) $4x^2y^2 + 5xy - 3xy^2$ $+ 6x^2y^2 + 8xy + 3xy^2$
$3x^2 - 2x - 3$	$3a^3 + 7ab + 5b^2$	$10x^2y^2 + 13xy$

Subtracting Polynomials

Distribute the negative sign, and then combine like terms.

4) $(2x^4 + x^3 - 4) - (x^4 - 4x^3 + 1)$ $-x^4 + 4x^3 - 1$	5) $(5y^2 + 8 + 7y) - (2y^2 - 4y + 3)$ $-2y^2 + 4y - 3$	7) $11m^2 + 2m - 11$ $-5m^2 + 6m + 17$
$x^4 + 5x^3 - 5$	$3y^2 + 11y + 5$	$6m^2 + 8m + 6$

Multiplying Polynomials

1. Expand (if there is an exponent)
2. Distribute 1st parenthesis to 2nd parenthesis
3. Repeat step 2 until all parentheses are gone
4. Combine like terms

8) $(2a + 7)^2$	OYO: $(5b - 3)(2b + 1)$
$(2a+7)(2a+7)$ $4a^2 + 14a + 14a + 49$ $4a^2 + 28a + 49$	$10b^2 - b - 3$

9) $(x^2 + 2x + 7)(x^2 + 5x - 9)$

$x^4 + 5x^3 - 9x^2 + 2x^3 + 10x^2 - 18x + 7x^2 + 35x - 63$

$x^4 + 7x^3 + 8x^2 - 17x - 63$

OYO: $(5x^2 - x - 4)(2x^2 + x + 12)$

$10x^4 + 3x^3 + 51x^2 - 16x - 48$

10) $(2x + 1)(3x - 2)(4x - 3)$

$6x^2 - 4x + 3x - 2$

$(6x^2 - x - 2)(4x - 3)$

$24x^3 - 18x^2 - 4x^2 + 3x - 8x + 6$

$24x^3 - 22x^2 - 5x + 6$

OYO: $(4x + 3)^3$

$64x^3 + 144x^2 + 108x + 27$

NOTES 10-2

ALGEBRA II UNIT 10 Polynomial Operations

Long Division

Remember long division: Divide → Multiply → Subtract → Bring Down

524 ÷ 5	1247 ÷ 3	2568 ÷ 10	324 ÷ 9
104 R4 5 | 524 −5↓ 02 −0↓ 24 −20 4	415 R2 3 | 1247 −12↓ 04 −3↓ 17 −15 2	256 R8 10 | 2568 −20↓ 56 −50↓ 68 −60 8	36 9 | 324 −27↓ 54 −54 0

Polynomial Long Division

Steps for Polynomial Long Division:

1. **Polynomial should be written with exponents in descending order**
2. **Write 0 as a place holder for any missing term**
3. **Divide (First divisor into first dividend)**
4. **Multiply (Answer times divisor--distribute)**
   -Write answer below dividend
5. **Subtract (change sign all terms)**
6. **Bring down next term from dividend**
7. **Repeat steps 1-4 until you have no terms left to bring down**

***If the remainder is zero, then we say that the binomial is a** factor **of the polynomial.**

1) $(6x^2+13x+8)\div(2x+3)$

$3x + 2 + \frac{2}{2x+3}$

$2x+3 \;\big|\; 6x^2+13x+8$

What's missing? $-6x^2 + 9x$

$4x + 8$

$-4x + 6$

2 remainder

$$3x + 2 + \frac{2}{2x+3}$$

2) $(15x^3+28x^2-4x-5)\div(3x+5)$

$5x^2 + x - 3 + \frac{10}{3x+5}$

$3x+5 \;\big|\; 15x^3+28x^2-4x-5$

$-15x^3 + -25x^2$ ↓

$3x^2 - 4x$

$-3x^2 + 5x$ ↓

$-9x - 5$

$+9x + 15$

10

$$5x^2 + x - 3 + \frac{10}{3x+5}$$

3) $(x^4-3x^3-7x-14)\div(x+2)$

$$x+2\;\overline{)\;x^4-3x^3+0x^2-7x-14}$$

$$x^3-5x^2+10x-27+\frac{40}{x+2}$$

$$-x^4+2x^3$$
$$-5x^3+0x^2$$
$$+5x^3+10x^2$$
$$10x^2-7x$$
$$-10x^2+20x$$
$$-27x-14$$
$$+27x+54$$
$$40$$

$$x^3-5x^2+10x-27+\frac{40}{x+2}$$

4) $(3x^3-11x^2-76x+48)\div(x+5)$

$$x+5\;\overline{)\;3x^3-11x^2-76x+48}$$

$$3x^2-26x+54+\frac{-222}{x+5}$$

$$-3x^3+15x^2$$
$$-26x^2-76x$$
$$+26x^2+130x$$
$$54x+48$$
$$-54x+270$$
$$-222$$

$$3x^2-26x+54+\frac{-222}{x+5}$$

5) $(2x^4+4x^3-5x^2+3x-2)\div(x^2+2x-3)$

$$x^2+2x-3\;\overline{)\;2x^4+4x^3-5x^2+3x-2}$$

$$2x^2+1+\frac{x+1}{x^2+2x-3}$$

$$-2x^4+4x^3+6x^2$$
$$x^2+3x-2$$
$$-x^2+2x+3$$
$$x+1$$

$$2x^2+1+\frac{x+1}{x^2+2x-3}$$

6) $(x^4-3x^3-7x-14)\div(x+1)$

$$x+1\;\overline{)\;x^4-3x^3+0x^2-7x-14}$$

$$x^3-4x^2+4x-11+\frac{-3}{x+1}$$

$$-x^4+x^3$$
$$-4x^3+0x^2$$
$$+4x^3+4x^2$$
$$4x^2-7x$$
$$-4x^2+4x$$
$$-11x-14$$
$$+11x+11$$
$$-3$$

$$x^3-4x^2+4x-11+\frac{-3}{x+1}$$

Synthetic Division

Used to test if a number is a root/solution a given polynomial

Remainder = 0 ⟶ Number is a root

1. Polynomial should be written with exponents in **descending** order
2. Write the coefficients **only**
   - Put zero as a placeholder for any missing terms
3. Put the test root on the left (solve the factor when set equal to 0)
4. Carry down the first coefficient number
5. Multiply by the test root and place your answer up to the next column
6. Add down & write your answer below
7. Repeat steps 4 & 5 until there are no numbers left
8. In the answer, the numbers from RIGHT TO LEFT represent: Remainder, constant, x term, x^2 term, etc.

1) $(6x^3 + x^2 - 10x + 3) \div (x - 1)$ $x-1=0$, $x=1$

1 | 6 1 -10 3
↓ 6 7 -3
6x² 7x -3 | 0R

$6x^2 + 7x - 3$

2) $(x^3 - 10x - 12) \div (x + 2)$

-2 | 1 0 -10 -12
↓ -2 4 12
1x² -2x -6 | 0

$x^2 - 2x - 6$

3) $(x^3 - 2x^2 - 7x - 4) \div (x + 1)$

-1 | 1 -2 -7 -4
↓ -1 3 4
1x² -3x -4 | 0

$x^2 - 3x - 4$

4) $(x^4 - 4x^3 - 7x^2 + 34x - 20) \div (x + 3)$

-3 | 1 -4 -7 34 -20
↓ -3 21 -42 24
1x³ -7x² 14x -8 | 4

$x^3 - 7x^2 + 14x - 8 + \frac{4}{x^4 - 4x^3 - 7x^2 + 34x - 20}$

5) $(6x^4 - 13x^3 - 19x^2 + 12x) \div (x - 3)$

3 | 6 -13 -19 12 0
↓ 18 15 -12 0
6x³ 5x² -4x 0 | 0

$6x^3 + 5x^2 - 4x$

6) $(x^4 - 3x^3 + 2x^2 + 2x - 10) \div (x - 2)$

2 | 1 -3 2 2 -10
↓ 2 -2 0 4
1x³ -1x² 0x 2 | -6

$x^3 - x^2 + 2 + \frac{-6}{x-2}$

NOTES 10-3

ALGEBRA II UNIT 10 Polynomial Operations

Review Factoring:

1. Always factor out a GCF if possible
2. Find target product and target sum
3. Split the middle term
4. Factor by grouping (or shortcut if a=1)

1. $x^2 - 6x + 8$

8 | −6 (X | +)

$-4 \cdot -2 \mid -4 + -2$

$(x-4)(x-2)$

2. $6x^2 + 5x - 6$

$-36 \mid 5$ (X | +)

$9 \cdot -4 \mid 9 + -4$

$6x^2 + 9x - 4x - 6$

$3x \qquad -2$

$3x(2x+3) - 2(2x+3)$

$(2x+3)(3x-2)$

3. $x^2 - 7x + 10$

$(x-5)(x-2)$

10 | −7 (X | +)

$-5 \cdot -2 \mid -5 + -2$

4. $\frac{10x^2 - 22x + 4}{2}$

$2(5x^2 - 11x + 2)$

10 | −11 (X | +)

$-10 \cdot -1 \mid -10 + -1$

$5x^2 - 10x - 1x + 2$

$5x \qquad -1$

$5x(x-2) - 1(x-2)$

$2(x-2)(5x-1)$

5. $x^2 - 3x - 18$

$(x-6)(x+3)$

$-18 \mid -3$ (X | +)

$-6 \cdot 3 \mid -6 + 3$

6. $6x^2 + 19x + 10$

$6x^2 + 15x + 4x + 10$

$3x \qquad 2$

60 | 19 (X | +)

$15 \cdot 4 \mid 15 + 4$

$3x(2x+5) + 2(2x+5)$

$(2x+5)(3x+2)$

Use synthetic division with the given factor to simplify to a quadratic. Factor the quadratic to find the remaining factors. List all factors, including those given.

1) $2x^3 + 5x^2 - 14x - 8 = 0$ Factor: (x - 2)

2 | 2 5 −14 −8

↓ 4 18 8

$2x^2$ 9x 4 | 0

$2x^2 + 9x + 4$

$2x^2 + 8x + 1x + 4$

$2x \qquad 1$

$2x(x+4) + 1(x+4)$

8 | 9 (X | +)

$8 \cdot 1 \mid 8 + 1$

Factors: $(x+4)(2x+1)(x-2)$

2) $x^3 - 2x^2 - 7x - 4 = 0$ Factor: (x + 1)

−1 | 1 −2 −7 −4

↓ −1 3 4

$1x^2$ −3x −4 | 0

$x^2 - 3x - 4$

$(x-4)(x+1)$

−4 | −3 (X | +)

$-4 \cdot 1 \mid -4 + 1$

Factors: $(x-4)(x+1)(x+1)$

OR

$(x-4)(x+1)^2$

3) $6x^3 + x^2 - 10x + 3 = 0$ Factor: (x - 1)

1	6	1	-10	3
	↓	6	7	3
	$6x^2$	7x	-3	0

$6x^2 + 7x - 3$

$6x^2 + 9x - 2x - 3$

$3x$ $\quad$ -1

$3x(2x+3) - 1(2x+3)$

-18	7
X	+
$9 \cdot -2$	$9 + -2$

Factors: $(2x+3)(3x-1)(x-1)$

4) $x^3 - 10x + 12 = 0$ Factor: (x - 2)

2	1	0	-10	12
	↓	2	4	-12
	$1x^2$	2x	-6	0

$x^2 + 2x - 6$

-6	2
X	+

Can't factor

(1 linear, 1 quadratic)

Factors: $(x-2)(x^2+2x-6)$

5) $x^4 - 4x^3 - 7x^2 + 34x - 24 = 0$

Factors: (x + 3); (x – 1)

-3	1	-4	-7	34	-24
	↓	-3	21	-42	24
1	1	-7	14	-8	0
	↓	1	-6	8	
	$1x^2$	-6x	8	0	

$x^2 - 6x + 8$

$(x-4)(x-2)$

8	-6
X	+
$-4 \cdot -2$	$-4 + -2$

Factors: $(x-4)(x-2)(x+3)(x-1)$

6) $x^4 - 3x^3 + 2x^2 + 2x - 4 = 0$

Factors: (x + 1); (x – 2)

-1	1	-3	2	2	-4
	↓	-1	4	-6	4
2	1	-4	6	-4	0
	↓	2	-4	4	
	$1x^2$	-2x	2	0	

$x^2 - 2x + 2$

2	-2
X	+

can't factor

(2 linear, 1 quadratic)

Factors: $(x+1)(x-2)(x^2-2x+2)$

Perfect cubes:

$1^3 = 1$	$2^3 = 8$	$3^3 = 27$	$4^3 = 64$	$5^3 = 125$	$6^3 = 216$	$7^3 = 343$	$8^3 = 512$
$9^3 = 729$	$10^3 = 1000$	$11^3 = 1331$	$12^3 = 1728$	$13^3 = 2197$	$14^3 = 2744$	$15^3 = 3375$	$16^3 = 4096$

Factoring Sums of CUBES: $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

Factoring Differences of CUBES: $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

(SOAP: Same, Opposite, Always positive)

FACTORING a sum or difference of two cubes:

Step 1: Rewrite both numbers as perfect cubes. Place the bases in parentheses with the same sign.

Step 2: Square each of the terms from Step 1 and place them in the 1st & last term positions. Leave the middle term open.

Step 3: Multiply the bases by each other. Change the sign. This is your middle term.

1) $a^3 + 64$

$a^3 + 4^3$

$(a + 4)(a^2 - 4a + 16)$

$4a$

a^2 $\quad$ 4^2

2) $27u^3 - 125$

$(3u)^3 - 5^3$

$(3u - 5)(9u^2 + 15u + 25)$

$-15u$

$(3u)^2$ $\quad$ $(-5)^2$

3) $\frac{250x^4 + 128x}{2x}$ *look for a GCF

$2x(125x^3 + 64)$

$(5x)^3 + 4^3$

$2x(5x + 4)(25x^2 - 20x + 16)$

$20x$ $(5x)^2$ 4^2

4) $x^3 - 216y^3$

$x^3 - (6y)^3$

$(x - 6y)(x^2 + 6xy + 36y^2)$

$-6xy$

5) $125 - x^3$

$(5 - x)(25 + 5x + x^2)$

6) $\frac{-a^3 - 8}{-1}$ *factor out the negative

$-1(a^3 - 8)$

$-(a - 2)(a^2 + 2a + 4)$

7) $\frac{343x^3y + 216y^4}{y}$

$y(343x^3 + 216y^3)$

$(7x)^3 + (6y)^3$

$y(7x + 6y)(49x^2 - 42xy + 36y^2)$