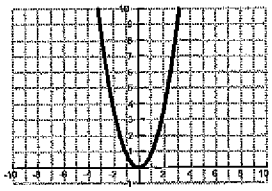
NOTES 1-1

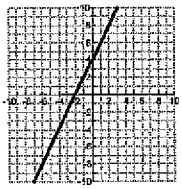
ALGEBRA II UNIT 1 Functions and Transformations

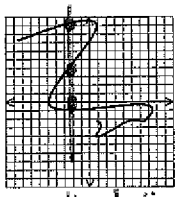
Determine if each relation is a Function or not.

GRAPH: the graph must pass the vertical line test **(meaning no vertical line would cross the graph in more than one place.**

1. Function or Not a Function 2. Function or Not a Function 3. Function or Not a Function 4. Function or Not a Function







vertical line passes through more than once

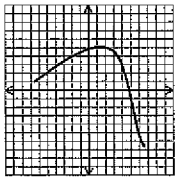


TABLE: the x-values **cannot repeat themselves (unless the x is paired with the same y).**

5. Function or Not a Function 6. Function or Not a Function 7. Function or Not a Function 8. Function or Not a Function

X	Y
-1	1
0	0
1	1
2	4
-1	1

*same ordered pair

X	Y
-3	5
-2	0
-2	-3
0	-4
1	-3

x = -2 repeats

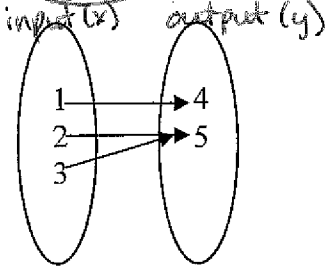
X	Y
-3	5
-2	5
-1	5
0	5
1	5

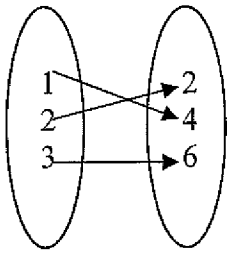
y's can repeat

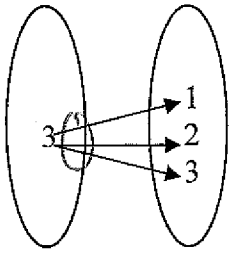
X	Y
-3	5
-2	0
-1	-3
0	-4
1	-3

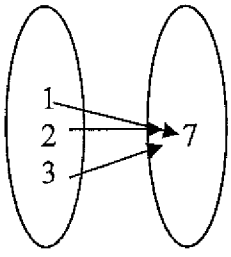
MAPPING: no input **can be used more than once (no more than 1 arrow coming from each input/x value).**

9. Function or Not a Function 10. Function or Not a Function 11. Function or Not a Function 12. Function or Not a Function









ORDERED PAIRS: the x values **cannot be repeated (unless the x is paired with the same y).**

13. Function or Not a Function

{(2, 4), (1, 4), (-1,4), (2, -3)}

14. Function or Not a Function

{(2, 4), (1, 9), (-1,9), (3, -3)}

15. Function or Not a Function

{(2, 4), (4, 2), (-1,9), (-3, 2)}

16. Function or Not a Function

{(2, 4), (1, 7), (-1,9), (3, -3)}

NOTES 1-2

ALGEBRA II UNIT 1 Functions and Transformations

Determining the Domain and Range: Always list the domain and range values from least **to** greatest.

Domain

The independent variable or input.

It is the x coordinate.

Look at the graph from left to right, along the x-axis.

X	Y
D	R
I	D

Range

The dependent variable or output.

It is the y coordinate.

Look at the graph from bottom to top, along the y-axis.

Interval Notation:

Domain: (#, ∞) **Use (for arrow or open circle)** **Range:** (-∞, ∞)

[#, #] **Use [for closed circle]** [#, #]

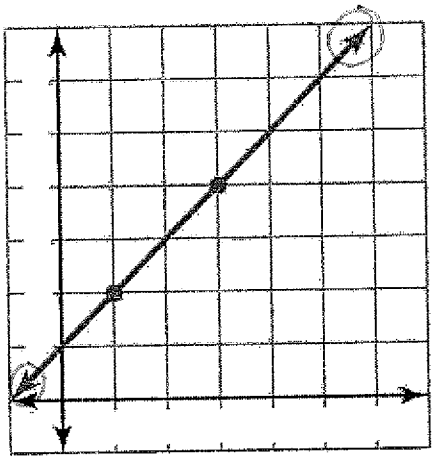
(-∞, ∞) → ℝ

Set Builder Notation:

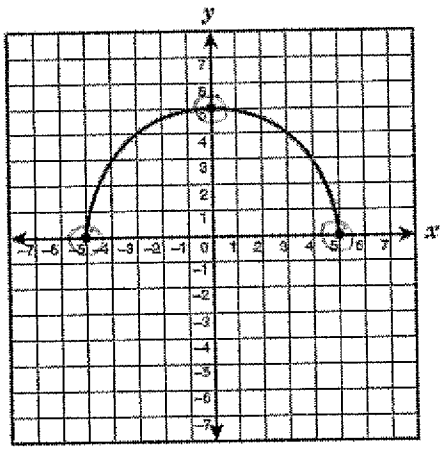
Domain: $\{x|-\infty < x < \infty\}$ **(Use < for arrow or open circle)** **Range:** $\{y|-\infty < y < \infty\}$

$\{x|\# \leq x \leq \#\}$ **(Use ≤ for closed circle)** $\{y|\# \leq y \leq \#\}$

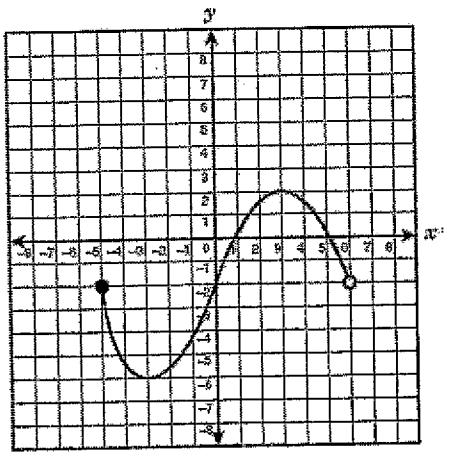
Define the domain and range using interval notation and using set builder notation.

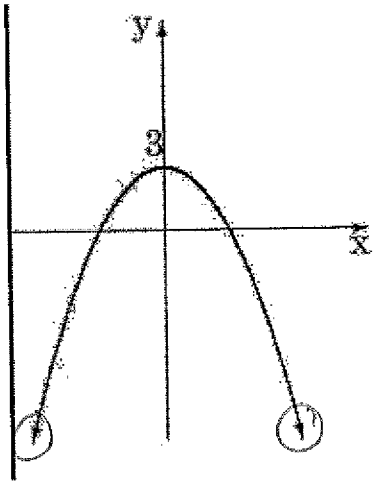


	Domain (x)	Range (y)
Interval:	(-∞, ∞)	(-∞, ∞)
Set:	$\{x\|-\infty < x < \infty\}$	$\{y\|-\infty < y < \infty\}$



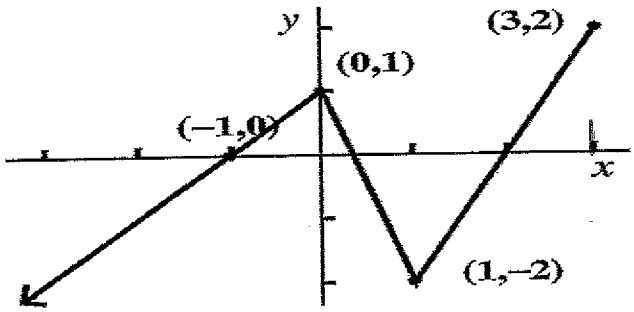
	Domain	Range
Interval:	[-5, 5]	[0, 5]
Set:	$\{x\|-5 \leq x \leq 5\}$	$\{y\|0 \leq y \leq 5\}$





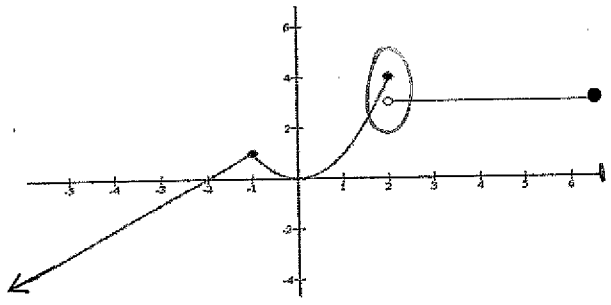
	Domain	Range
Interval:	$[-5, 6)$	$[-6, 2]$
Set:	$\{x \mid -5 \leq x < 6\}$	$\{y \mid -6 \leq y \leq 2\}$

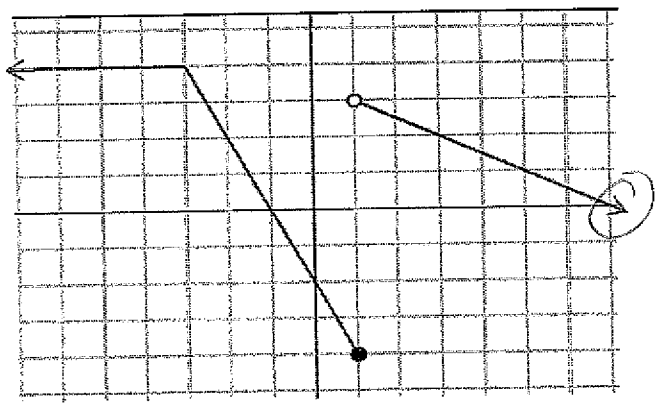
	Domain	Range
Interval:	$(-\infty, \infty)$	$(-\infty, 3]$
Set:	$\{x \mid -\infty < x < \infty\}$	$\{y \mid -\infty < y \leq 3\}$



	Domain	Range
Interval:	$(-\infty, 3]$	$(-\infty, 2]$
Set:	$\{x \mid -\infty < x \leq 3\}$	$\{y \mid -\infty < y \leq 2\}$

Piece-Wise Functions: Functions composed of multiple pieces





	Domain	Range
Interval:	$(-\infty, 7]$	$(-\infty, 4]$
Set:	$\{x \mid -\infty < x \leq 7\}$	$\{y \mid -\infty < y \leq 4\}$

	Domain	Range
Interval:	$(-\infty, \infty)$	$(-\infty, 4]$
Set:	$\{x \mid -\infty < x < \infty\}$	$\{y \mid -\infty < y \leq 4\}$

Algebra II Unit 1 Functions and Transformations **NOTES 1-3**

TRANSFORMATIONS: Consider the function $f(x)$

Vertical Stretch/ Compression or Reflect Vertically:

Suppose we multiply the function *f(x)* by **a:** $f(x) \rightarrow a \cdot f(x)$

The value of ***a*** affects the vertical stretch or compression of a function.

The sign of ***a*** tells you whether the function reflects vertically.

*If $a > 1$, then the function is **stretched** or becomes **"skinnier/steeper"**. *If $0 < a < 1$, then the function is **compressed** or becomes **"wider/more flat"**. *If the sign of ***a*** is negative, then the function reflects vertically.

Examples: Describe the transformation:

1. $f(x) \rightarrow 3f(x)$ V. Stretch of 3
2. $h(x) \rightarrow -h(x)$ V. Reflection
3. $g(x) \rightarrow -\frac{1}{2}g(x)$ V. Reflection V. Compression of $\frac{1}{2}$

OYO:

1. $f(x) \rightarrow -2f(x)$
2. $h(x) \rightarrow \frac{3}{4}h(x)$
3. $g(x) \rightarrow 6g(x)$

Horizontal Stretch/ Compression or Reflect Horizontally:

Suppose we multiply x by **b** on the inside the *f(x)*: $f(x) \rightarrow f[b(x)]$

The value of b effects the horizontal stretch or compression of *f(x)*.

The sign of b tells whether or not the function reflects horizontally.

WATCH OUT: When writing the transformation you must write $\frac{1}{b}$ (the reciprocal of *b*). *If $\frac{1}{b} > 1$ then it is a horizontal stretch, if $0 < \frac{1}{b} < 1$ then it is a horizontal compression.*

*If f[2(x)] then the function has a horizontal *compression* of ½.

*If f[½ x)] then the has a horizontal *stretch* of 2.

Examples: Describe the transformations:

* "b" must be factored out in order to describe the horizontal shift correctly. If you do not factor " b" out your horizontal shift will be wrong!

1. $f(x) \rightarrow 2f[\frac{1}{3}(x)]$ $\frac{1}{3} \rightarrow 3$ V. Stretch of 2 H. Stretch of 3
2. $h(x) \rightarrow -h[-5(x)]$ $5 \rightarrow \frac{1}{5}$ V. Reflect H. Reflect H. Compress of $\frac{1}{5}$
3. $g(x) \rightarrow -3g[\frac{1}{2}(x)]$ $\frac{1}{2} \rightarrow 2$ V. Reflect V. Stretch of 3 H. Stretch of 2

OYO:

1. $f(x) \rightarrow -f[4(x)]$
2. $h(x) \rightarrow \frac{1}{2}h[\frac{1}{5}(x)]$
3. $g(x) \rightarrow 7g[\frac{1}{4}(x)]$

Horizontal Translation: Suppose we subtract *h* from x inside the function f(x): f(x) → f(x - h)

The value of ***h*** effects the horizontal shift of f(x). The sign of ***h*** tells whether the function moves left or right.

WATCH OUT *If f(x + h) then the function shifts **left**. *If f(x - h) then the function shifts **right**.

Examples: Describe the transformations:

1. $f(x) \to f[2(x+5)]$ $x-(-5)$
   2 → $\frac{1}{2}$ H. Comp of $\frac{1}{2}$
   Shift left 5
2. $h(x) \to -h(x-7)$
   V. Reflect
   Shift Right 7
3. $g(x) \to -\frac{1}{2}g(5x+15)$ $[5(x+3)]$
   V. Reflect
   V. Comp of $\frac{1}{2}$
   5 → $\frac{1}{5}$ H. Comp of $\frac{1}{5}$
   Shift left 3

OYO:

1. $f(x) \to -f(x-2)$
2. $h(x) \to 2h(x+3)$
3. $g(x) \to -g(3x-12)$

Vertical Translation: Suppose we add *k* to the function *f(x)*: f(x) → f(x) + k

The value of ***k*** effects the vertical shift of the function.

The sign of ***k*** tells whether the function moves up or down.

*If we add ***k***, then the function moves **up**. *If we subtract ***k***, then the function moves **down**.

Examples: Describe the transformations:

1. $f(x) \to f[2(x-3)] - 9$
   2 → $\frac{1}{2}$ H. Comp of $\frac{1}{2}$
   Shift right 3
   Shift down 9
2. $h(x) \to 3h(2x-4) - 5$ $[2(x-2)]$
   V. Stretch of 3
   2 → $\frac{1}{2}$ H. Comp of $\frac{1}{2}$
   Shift right 2
   Shift down 5
3. $g(x) \to -\frac{1}{2}g\left(\frac{1}{3}x\right) + 2$
   V. Reflect
   V. Comp of $\frac{1}{2}$
   $\frac{1}{3}$ → 3 H. Stretch of 3
   Shift up 2

OYO:

1. $f(x) \to \frac{1}{2}f(x+4) - 7$
2. $h(x) \to -h(4x-8) + 5$
3. $g(x) \to 5g(x+2) - 8$

List the transformations: $f(x) \rightarrow \overset{a}{3} f\left[\overset{b}{\frac{1}{2}}(x - \overset{h}{4})\right] + \overset{k}{5}$

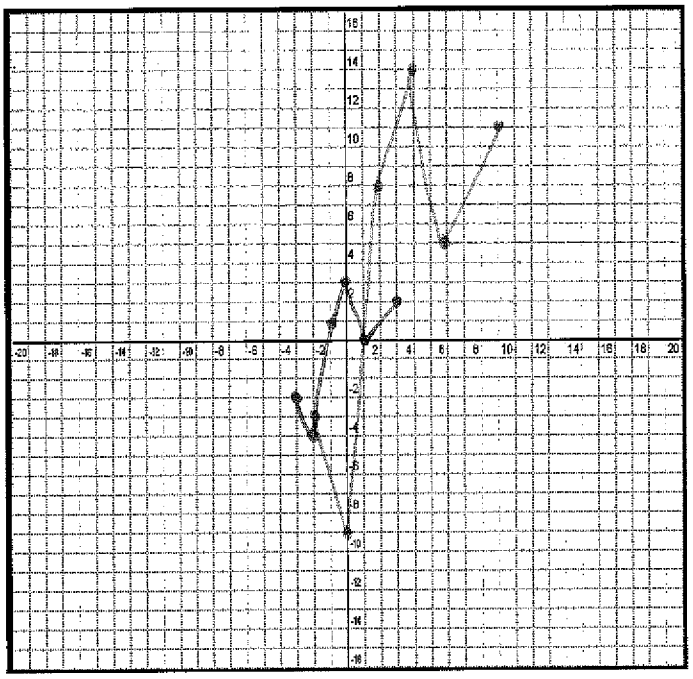
a 1. V. Stretch of 3 *b 2.* H. Stretch of 2 *h 3.* Shift right 4 *k 4.* Shift up 5

Complete the table, graph the original function, then graph the transformed function:

h +4 ±	1/b * 2 ( )	x	y	a * (3)	k +5 ±
-2	-6	-3	-3	-9	-4
0	-4	-2	-5	-15	-10
2	-2	-1	1	3	8
4	0	0	3	9	14
6	2	1	0	0	5
10	6	3	2	6	11

new x's

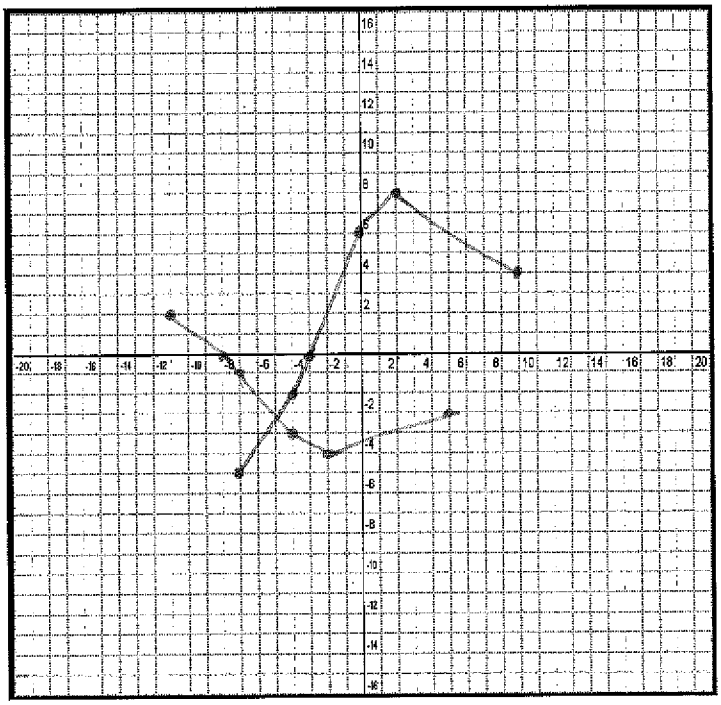
new y's



List the transformations: $g(x) \rightarrow -\frac{1}{2} g(x + 4) - 1$

a 1. V. Reflection *h 3.* Shift left 4

a 2. V. Compress of $\frac{1}{2}$ *k 4.* Shift down 1



Complete the table, graph the original function, then graph the transformed function:

h -4 ±	1/b * (1)	x	y	a * ($-\frac{1}{2}$)	k -1 ±
-11	-7	-7	-6	3	2
-8	-4	-4	-2	1	0
-7	-3	-3	0	0	-1
-4	0	0	6	-3	-4
-2	2	2	8	-4	-5
5	9	9	4	-2	-3