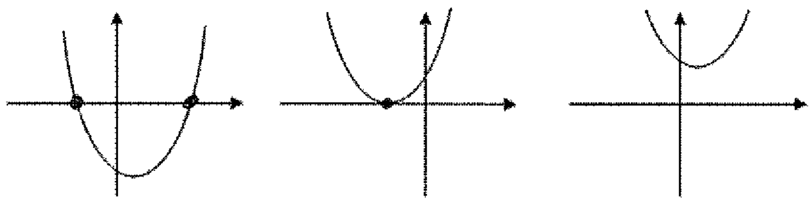
NOTES 5-1

Algebra II UNIT 5 Complex Numbers

In Algebra I:

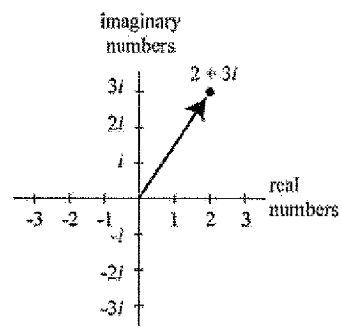
Solving a quadratic function, gives us the roots (zeros) of a parabola.



two roots | one root | no roots

$x^2 - 4 = 0$ | $x^2 = 0$ | $x^2 + 4 = 0$

In Algebra II, we introduce …



__Imaginary Numbers:__ The square root of a negative number

$\sqrt{-1} = i$

__Complex Numbers:__ A combination of a real number and an imaginary number.

Complex numbers are used to describe zeros (solutions) of quadratic functions that have no real roots.

__Simplifying radicals:__

1. Create a factor tree of the number under the radical
2. For each pair of factors (which together are a perfect square number), one can be brought out of the radical and is multiplied with the coefficient of the radical
3. $\sqrt{-1}$ can be brought out of the radical and is written as *i*.
4. The remaining factors, if any, under the radical are multiplied back together

1. $\sqrt{-1} = i$

2. $\sqrt{-4} =$

-1 4
2 2

$2i$

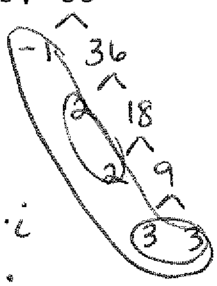
3. $\sqrt{-75} =$

-1 75
3 25
5 5

$5i$

TOGETHER

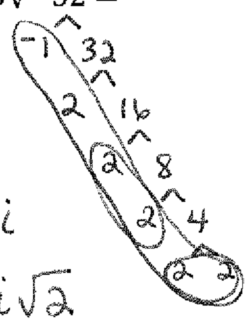
4. $-3\sqrt{-36} =$



$-3 \cdot 3 \cdot 2 \cdot i$

$-18i$

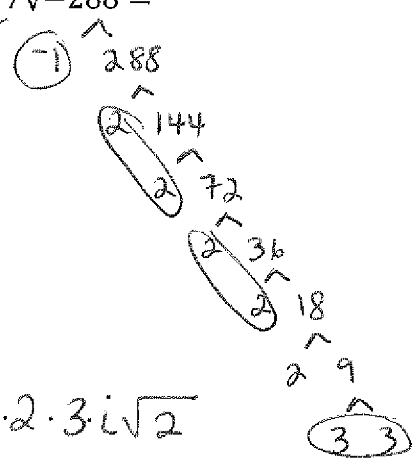
7. $5\sqrt{-32} =$



$5 \cdot 2 \cdot 2 \cdot i$

$20i\sqrt{2}$

9. $7\sqrt{-288} =$



$7 \cdot 2 \cdot 2 \cdot 3 \cdot i\sqrt{2}$

$84i\sqrt{2}$

OYO

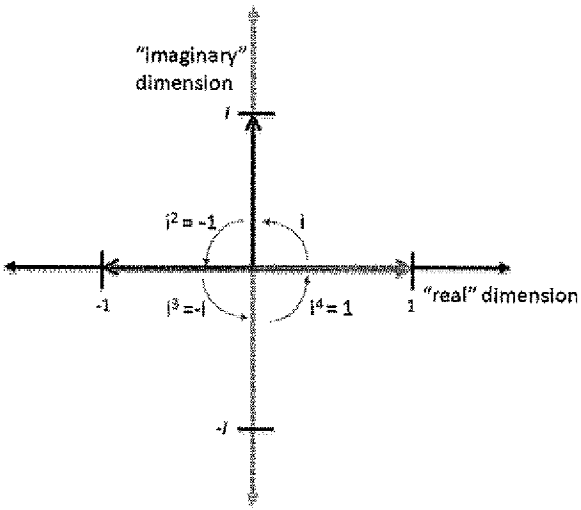
5. $-5\sqrt{-8} =$

8. $3\sqrt{-50} =$

10. $4\sqrt{-300}$

Algebra II UNIT 5 Complex numbers

Powers of Imaginary Numbers:



$i^0 = 1$

$i^1 = i$

$i^2 = -1$

$i^3 = -i$

$i^4 = 1$

$i^5 = i$

$i^6 = -1$

$i^7 = -i$

$i^8 = 1$

$i^9 = i$

$i^{10} = -1$

$i^{11} = -i$

$i^{12} = 1$

To find the value of i raised to a power:

- Divide the exponent by four → there are only 4 options
- Use the remainder to determine the answer

1. $i = i$

2. $i^{15} = -i$

   $15 \div 4 = 3$, $15 - 12 = 3$ → $i^3 = -i$

3. $i^4 = 1$

   $4 \div 4 = 1$, $4 - 4 = 0$ → $i^0 = 1$

4. $i^{17} = i$

   $17 \div 4 = 4$, $17 - 16 = 1$ → $i^1 = i$

5. $i^6 = -1$

   $6 \div 4 = 1$, $6 - 4 = 2$ → $i^2 = -1$

6. $i^{18} = -1$

   $18 \div 4 = 4$, $18 - 16 = 2$ → $i^2 = -1$

7. $i^{2320} = 1$

   $2320 \div 4 = 58$: -20, 32, -32, 00 → $i^0 = 1$

8. $i^{6533} = i$

   $6533 \div 4 = 1633$: -4, 25, -24, 13, -12, 13, -12, 1 → $i^1 = i$

9. $i^{375} = -1$

   $375 \div 4 = 93$: -36, 15, -12, 3 → $i^3 = -1$

NOTES 5-3

Algebra II UNIT 5 Complex numbers

Adding and Subtracting Complex Numbers:

Let's review adding and subtracting algebraic expressions:

$x + 6x + 12$

$7x + 12$

$1 - 8x - 4 - x$

$-3 - 9x$

$7 + 6x - 2(-5x + 3) - 8$

$7 + 6x + 10x - 6 - 8$

$-7 + 16x$

Combine like terms → imaginary term with imaginary terms
→ Real terms with real terms.

1. $i + 6i + 12$

$7i + 12$

2. $1 - 8i - 4 - i$

$-3 - 9i$

3. $7 + 6i - 2(-5i + 3) - 8$

$7 + 6i + 10i - 6 - 8$

$-7 + 16i$

4. $(19 - 12i) - (9 - 4i)$

$19 - 12i - 9 + 4i$

$10 - 8i$

5. $(2 + 7i) + (3 - 5i)$

$5 + 2i$

Multiplying Complex Numbers:

Distribute/FOIL → When we multiply variables, we add their exponents.
→ Remember, $i^2 = -1$

Let's review multiplying (distributing) algebraic expressions:

$7x(5 - 7x)$

$35x - 49x^2$

$(7 + 6x)^2$

$(7 + 6x)(7 + 6x)$

$49 + 42x + 42x + 36x^2$

$49 + 84x + 36x^2$

$(3 - 8x)(7 + 9x)$

$21 + 27x - 56x - 72x^2$

$21 - 29x - 72x^2$

6. $7i(5 - 7i)$

$35i - 49i^2$

$35i - 49(-1)$

$35i + 49$

7. $(7 + 6i)^2$

$(7 + 6i)(7 + 6i)$

$49 + 42i + 42i + 36i^2$

$49 + 84i + 36i^2$

$49 + 84i + 36(-1)$

$49 + 84i - 36$

$13 + 84i$

8. $(3 - 8i)(7 + 9i)$

$21 + 27i - 56i - 72i^2$

$21 - 29i - 72i^2$

$21 - 29i - 72(-1)$

$21 - 29i + 72$

$93 - 29i$

9. $(5-7i)^2$

$(5-7i)(5-7i)$

$25-35i-35i+49i^2$

$25-70i+49i^2$

$25-70i+49(-1)$

$25-70i-49$

$-24-70i$

10. $i(7+6i)$

$7i+6i^2$

$7i+6(-1)$

$7i-6$

Write the Complex Conjugate, Then MULTIPLY by the complex conjugate

Complex Conjugate: the complex conjugate of $a+bi$ is $\underline{a}\ \underline{-}\ \underline{bi}$.
Everything remains the same except for the ___sign___ of the ___imaginary___ term.

11. $2-6i$

Complex Conjugate: $2+6i$

Multiply:

$(2-6i)(2+6i)$

$4+\cancel{12i}-\cancel{12i}-36i^2$

$4-36(-1)$

$4+36$

40

12. $8-5i$

Complex Conjugate: $8+5i$

Multiply:

$(8-5i)(8+5i)$

$64+\cancel{40i}-\cancel{40i}-25i^2$

$64-25(-1)$

$64+25$

89

13. $3+9i$

Complex Conjugate: $3-9i$

Multiply:

$(3+9i)(3-9i)$

$9-\cancel{27i}+\cancel{27i}-81i^2$

$9-81(-1)$

$9+81$

90

14. $6+12i$

Complex Conjugate: $6-12i$

Multiply:

$(6+12i)(6-12i)$

$36-\cancel{72i}+\cancel{72i}-144i^2$

$36-144(-1)$

$36+144$

180